

Explanation for pore water pressure build-up process of sandy deposits due to seismic excitement using a numerical tool

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Stress is a tensor which can be represented by a matrix in Cartesian coordinates:

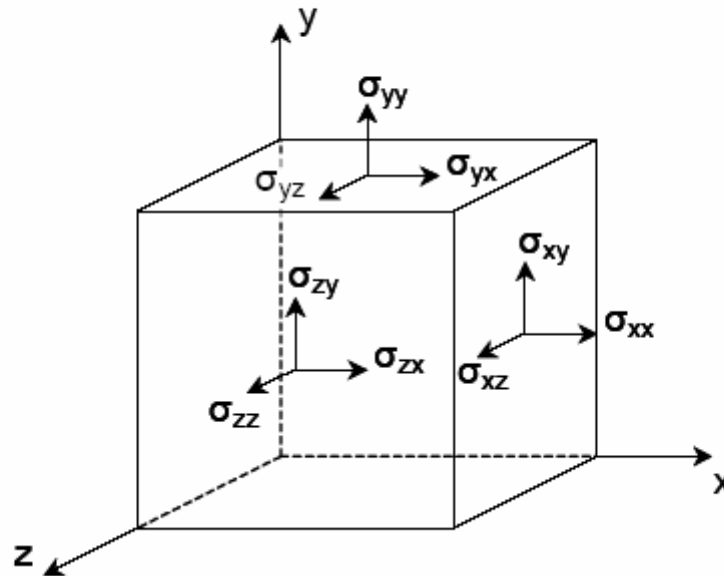
$$\underline{\underline{\sigma}} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

According to Terzaghi's principle, stresses in the soil are divided into effective stresses, σ' , and pore pressures, σ_w :

$$\underline{\underline{\sigma}} = \underline{\underline{\sigma'}} + \underline{\underline{\sigma}}_w$$

Material models for soil and rock are generally expressed as a relationship between infinitesimal increments of effective stress and infinitesimal increments of strain. In such a relationship, infinitesimal increments of effective stress are represented by stress rates (with a dot above the stress symbol):

$$\underline{\dot{\sigma}'} = (\dot{\sigma}'_{xx} \quad \dot{\sigma}'_{yy} \quad \dot{\sigma}'_{zz} \quad \dot{\sigma}'_{xy} \quad \dot{\sigma}'_{yz} \quad \dot{\sigma}'_{zx})^T$$



Strain is a tensor which can be represented by a matrix with Cartesian coordinates as:

$$\underline{\underline{\epsilon}} = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix}$$

According to the small deformation theory, only the sum of complementing Cartesian shear strain components ϵ_{ij} and ϵ_{ji} result in shear stress. This sum is denoted as the shear strain γ_{ij} . Hence, instead of ϵ_{xy} , ϵ_{yx} , ϵ_{yz} , ϵ_{zy} , ϵ_{zx} and ϵ_{xz} the shear strain components γ_{xy} , γ_{yz} and γ_{zx} are used respectively.

$$\underline{\varepsilon} = (\varepsilon_{xx} \quad \varepsilon_{yy} \quad \varepsilon_{zz} \quad \gamma_{xy} \quad \gamma_{yz} \quad \gamma_{zx})^T$$

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x}$$

$$\varepsilon_{yy} = \frac{\partial u_y}{\partial y}$$

$$\varepsilon_{zz} = \frac{\partial u_z}{\partial z}$$

$$\gamma_{xy} = \varepsilon_{xy} + \varepsilon_{yx} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}$$

$$\gamma_{yz} = \varepsilon_{yz} + \varepsilon_{zy} = \frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}$$

$$\gamma_{zx} = \varepsilon_{zx} + \varepsilon_{xz} = \frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}$$

In analogy to the invariants of stress, it is also useful to define invariants of strain. A strain invariant that is often used is the volumetric strain, ϵ_v , which is defined as the sum of all normal strain components:

$$\epsilon_v = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz} = \epsilon_1 + \epsilon_2 + \epsilon_3$$

The volumetric strain is defined as negative for compaction and as positive for dilatancy. For elastoplastic models strains are decomposed into elastic and plastic components:

$$\underline{\epsilon} = \underline{\epsilon}^e + \underline{\epsilon}^p$$

ELASTIC STRAINS

Material models for soil and rock are generally expressed as a relationship between infinitesimal increments of effective stress ('effective stress rates') and infinitesimal increments of strain ('strain rates'). This relationship may be expressed in the form:

$$\underline{\dot{\sigma}'} = \underline{\underline{M}} \underline{\dot{\epsilon}}$$

M is a material stiffness matrix. Note that in this type of approach, pore-pressures are explicitly excluded from the stress-strain relationship.

Hooke's law

$$\begin{bmatrix} \dot{\sigma}'_{xx} \\ \dot{\sigma}'_{yy} \\ \dot{\sigma}'_{zz} \\ \dot{\sigma}'_{xy} \\ \dot{\sigma}'_{yz} \\ \dot{\sigma}'_{zx} \end{bmatrix} = \frac{E}{(1-2\nu')(1+\nu')} \begin{bmatrix} 1-\nu' & \nu' & \nu' & 0 & 0 & 0 \\ \nu' & 1-\nu' & \nu' & 0 & 0 & 0 \\ \nu' & \nu' & 1-\nu' & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}-\nu' & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2}-\nu' & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\nu' \end{bmatrix} \begin{bmatrix} \dot{\epsilon}_{xx} \\ \dot{\epsilon}_{yy} \\ \dot{\epsilon}_{zz} \\ \dot{\gamma}_{xy} \\ \dot{\gamma}_{yz} \\ \dot{\gamma}_{zx} \end{bmatrix}$$

According to Terzaghi's principle, total stresses can be divided into effective stresses σ' and pore pressures σ_w .

$$\sigma_{xx} = \sigma'_{xx} + \sigma_w$$

$$\sigma_{yy} = \sigma'_{yy} + \sigma_w$$

$$\sigma_{zz} = \sigma'_{zz} + \sigma_w$$

$$\sigma_{xy} = \sigma'_{xy}$$

$$\sigma_{yz} = \sigma'_{yz}$$

$$\sigma_{zx} = \sigma'_{zx}$$

$$\sigma_w = p_{steady} + p_{excess}$$

Since the time derivative of the steady state component equals zero, it follows:

$$\dot{\sigma}_w = \dot{P}_{excess}$$

Hooke's law can be put as

$$\begin{bmatrix} \dot{\epsilon}_{xx}^e \\ \dot{\epsilon}_{yy}^e \\ \dot{\epsilon}_{zz}^e \\ \dot{\gamma}_{xy}^e \\ \dot{\gamma}_{yz}^e \\ \dot{\gamma}_{zx}^e \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu' & -\nu' & 0 & 0 & 0 \\ -\nu' & 1 & -\nu' & 0 & 0 & 0 \\ -\nu' & -\nu' & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2+2\nu' & 0 & 0 \\ 0 & 0 & 0 & 0 & 2+2\nu' & 0 \\ 0 & 0 & 0 & 0 & 0 & 2+2\nu' \end{bmatrix} \begin{bmatrix} \dot{\sigma}_{xx} - \dot{\sigma}_w \\ \dot{\sigma}_{yy} - \dot{\sigma}_w \\ \dot{\sigma}_{zz} - \dot{\sigma}_w \\ \dot{\sigma}_{xy} \\ \dot{\sigma}_{yz} \\ \dot{\sigma}_{zx} \end{bmatrix}$$

Considering slightly compressible water, the rate of pore pressure is written as:

$$\dot{\sigma}_w = \frac{K_w}{n} (\dot{\epsilon}_{xx}^e + \dot{\epsilon}_{yy}^e + \dot{\epsilon}_{zz}^e)$$

in which K_w is the bulk modulus of the water and n is the soil porosity.

Total stress: $\Delta p = K_u \Delta \epsilon_v$

Effective stress: $\Delta p' = (1 - B) \Delta p = K' \Delta \epsilon_v$

Excess pore pressure: $\Delta p_w = B \Delta p = \frac{K_w}{n} \Delta \epsilon_v$

$$\frac{K_w}{n} = \frac{K_w^0 K_{air}}{S K_{air} + (1 - S) K_w^0} \frac{1}{n} \quad \text{where} \quad K' = \frac{E'}{3(1 - 2\nu')}$$

Undrained parameters :

$$K_u = \frac{2G(1+\nu_u)}{3(1-2\nu_u)} \quad \text{where} \quad G = \frac{E'}{2(1+\nu')}$$

Also, it shall be mentioned

$$\nu_u = 0.495$$

$$\nu_u = \frac{3\nu' + B(1 - 2\nu')}{3 - B(1 - 2\nu')}$$

The value of Skempton's B parameter can now be calculated from the ratio of the bulk stiffnesses of the soil skeleton and the pore fluid:

$$B = \frac{1}{1 + \frac{nK'}{K_w}}$$

The rate of excess pore pressure is calculated from the (small) volumetric strain rate, according to:

$$\dot{\sigma}_w = \frac{K_w}{n} \dot{\epsilon}_v$$

The basic equation for the time-dependent movement of a volume under the influence of a (dynamic) load is:

$$\underline{\underline{M}} \underline{\underline{\ddot{u}}} + \underline{\underline{C}} \underline{\underline{\dot{u}}} + \underline{\underline{K}} \underline{\underline{u}} = \underline{\underline{F}}$$

M is the mass matrix, u is the displacement vector, C is the damping matrix, K is the stiffness matrix and F is the load vector.

The displacement, u , the velocity, \dot{u} , and the acceleration, \ddot{u} , can vary with time.

$$\underline{\underline{C}} = \alpha_R \underline{\underline{M}} + \beta_R \underline{\underline{K}}$$

$$u^{t+\Delta t} = u^t + \dot{u}^t \Delta t + \left(\left(\frac{1}{2} - \alpha \right) \ddot{u}^t + \alpha \ddot{u}^{t+\Delta t} \right) \Delta t^2$$

$$\dot{u}^{t+\Delta t} = \dot{u}^t + \left((1 - \beta) \ddot{u}^t + \beta \ddot{u}^{t+\Delta t} \right) \Delta t$$

Δt is the time step. The coefficients α and β determine the accuracy of the numerical time integration.

$$\beta \geq 0.5 \quad \alpha \geq \frac{1}{4} \left(\frac{1}{2} + \beta \right)^2$$

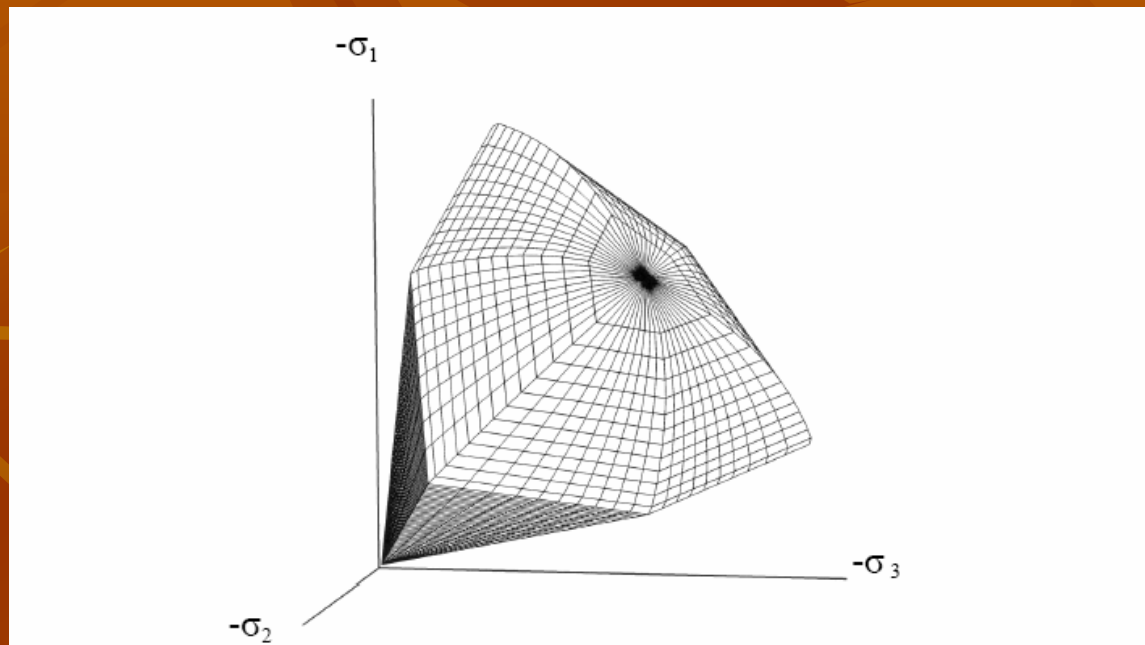
Soil Model

The Hardening-Soil model is an advanced model for simulating the behaviour of different types of soil, both soft soils and stiff soils, Schanz (1998). When subjected to primary deviatoric loading, soil shows a decreasing stiffness and simultaneously irreversible plastic strains develop. In the special case of a drained triaxial test, the observed relationship between the axial strain and the deviatoric stress can be well approximated by a hyperbola.

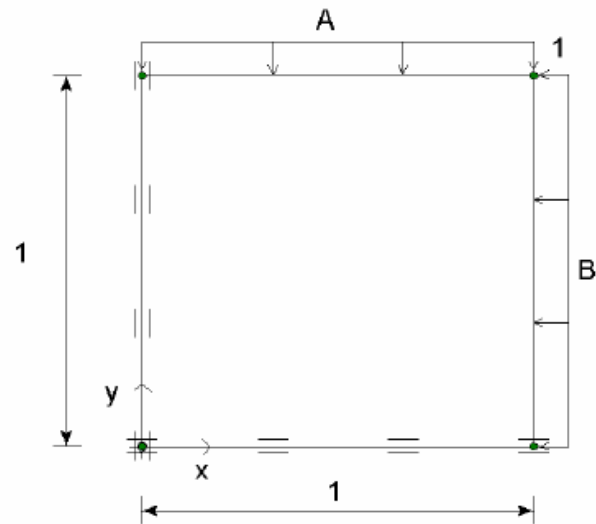
Firstly by using the theory of plasticity rather than the theory of elasticity.

Secondly by including soil dilatancy and thirdly by introducing a yield cap.

Stress dependent stiffness according to a power law.	Input parameter m
Plastic straining due to primary deviatoric loading.	Input parameter E_{50}^{ref}
Plastic straining due to primary compression.	Input parameter E_{oed}^{ref}
Elastic unloading / reloading.	Input parameters E_{ur}^{ref} , ν_{ur}
Failure according to the Mohr-Coulomb model.	Parameters c , φ and ψ

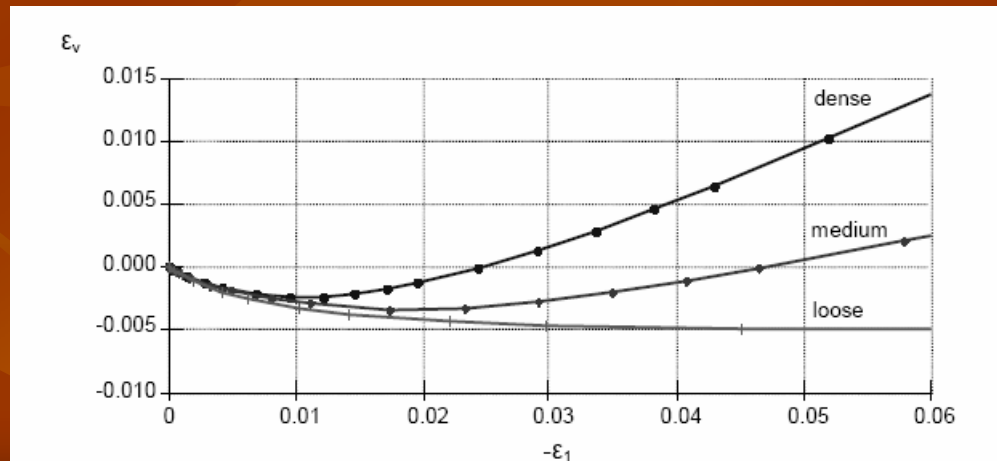
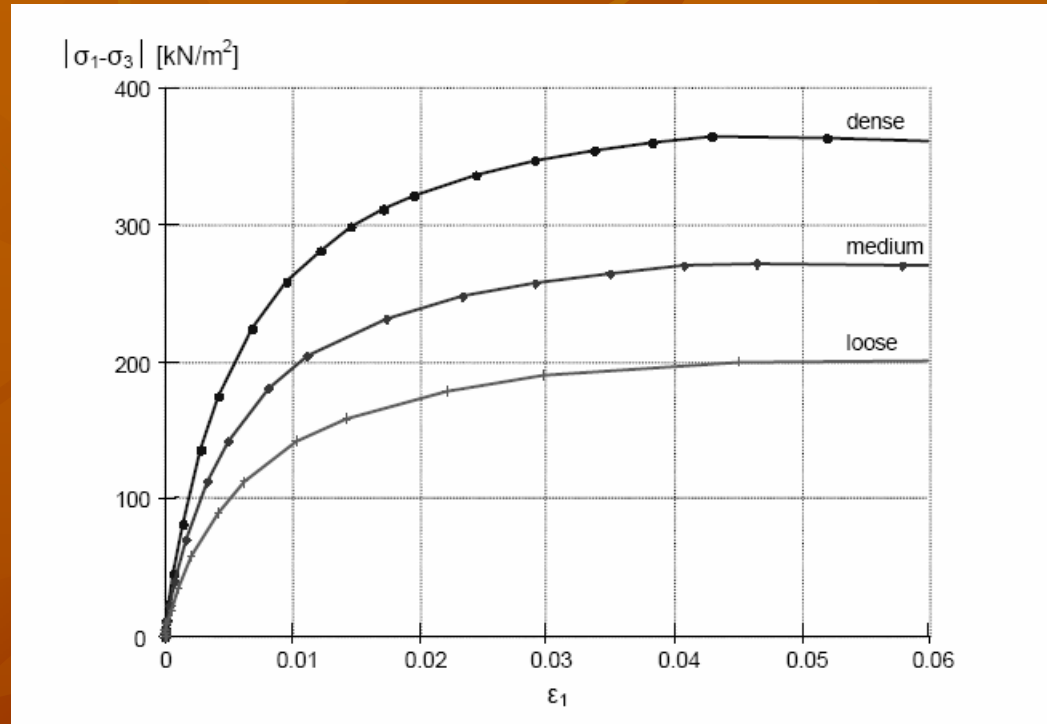


Representation of total yield contour of the Hardening-Soil model in principal stress space for cohesionless soil

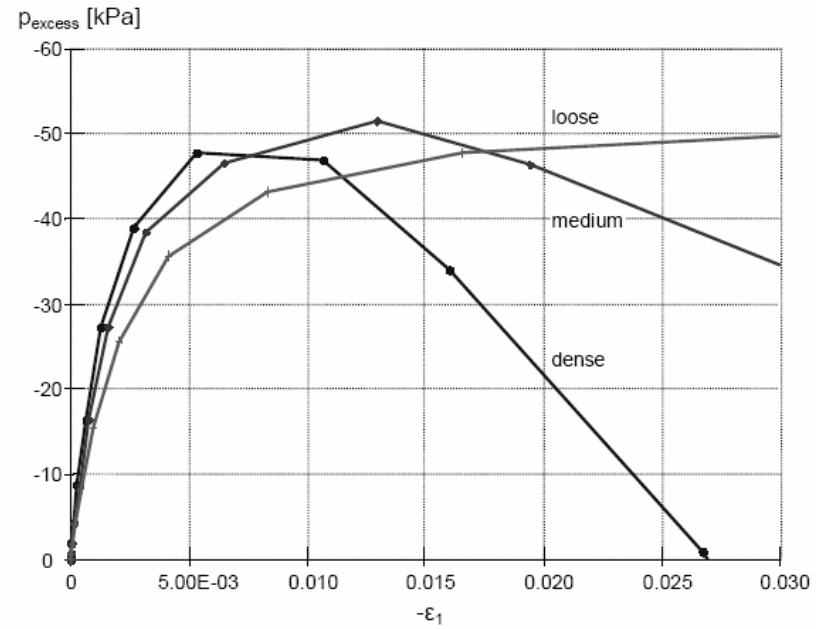
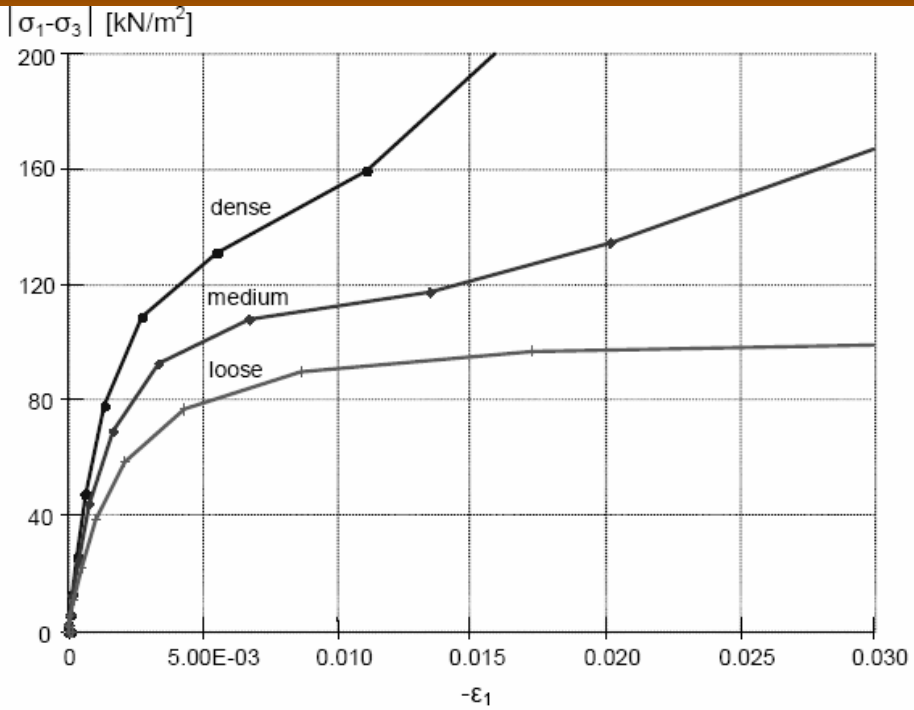


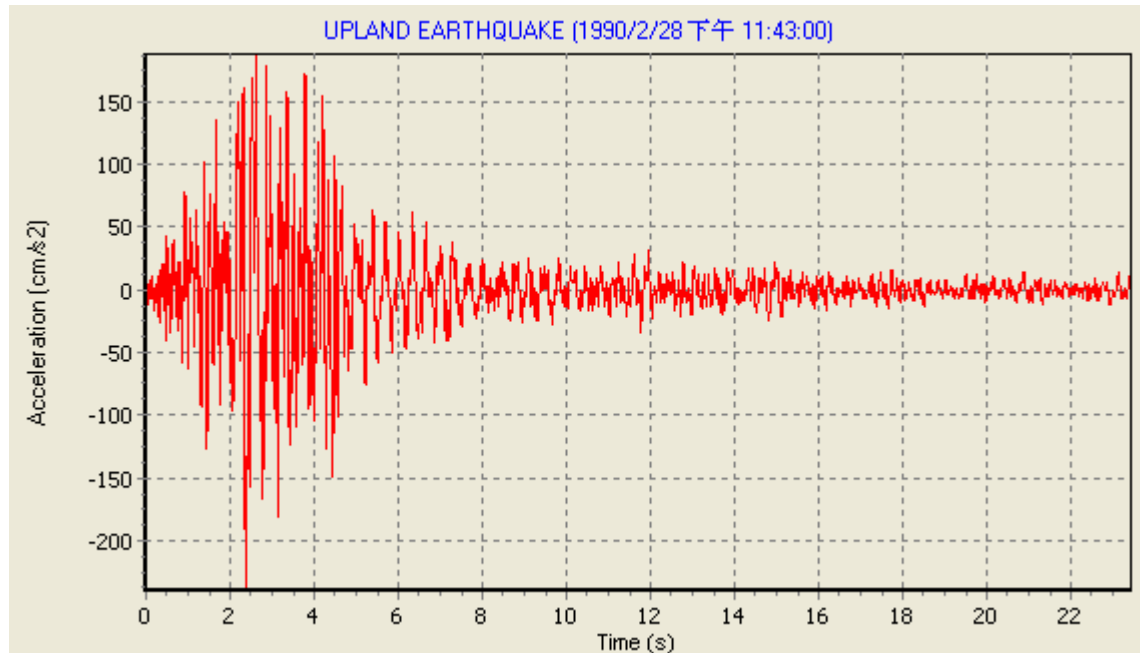
Parameter	Loose	Medium	Dense	Unit
E_{50}^{ref} (for $p_{ref} = 100$ kPa)	20000	30000	40000	kN/m ²
E_{ur}^{ref} (for $p_{ref} = 100$ kPa)	60000	90000	120000	kN/m ²
E_{oed}^{ref} (for $p_{ref} = 100$ kPa)	20000	30000	40000	kN/m ²
Cohesion c	0.0	0.0	0.0	kN/m ²
Friction angle φ	30	35	40	°
Dilatance angle ψ	0	5	10	°
Poisson's ratio ν_{ur}	0.2	0.2	0.2	-
Power m	0.5	0.5	0.5	-
K_0^{nc} (using Cap)	0.5	0.43	0.36	-
Tensile strength	0.0	0.0	0.0	kN/m ²
Failure ratio	0.9	0.9	0.9	-

Drained condition

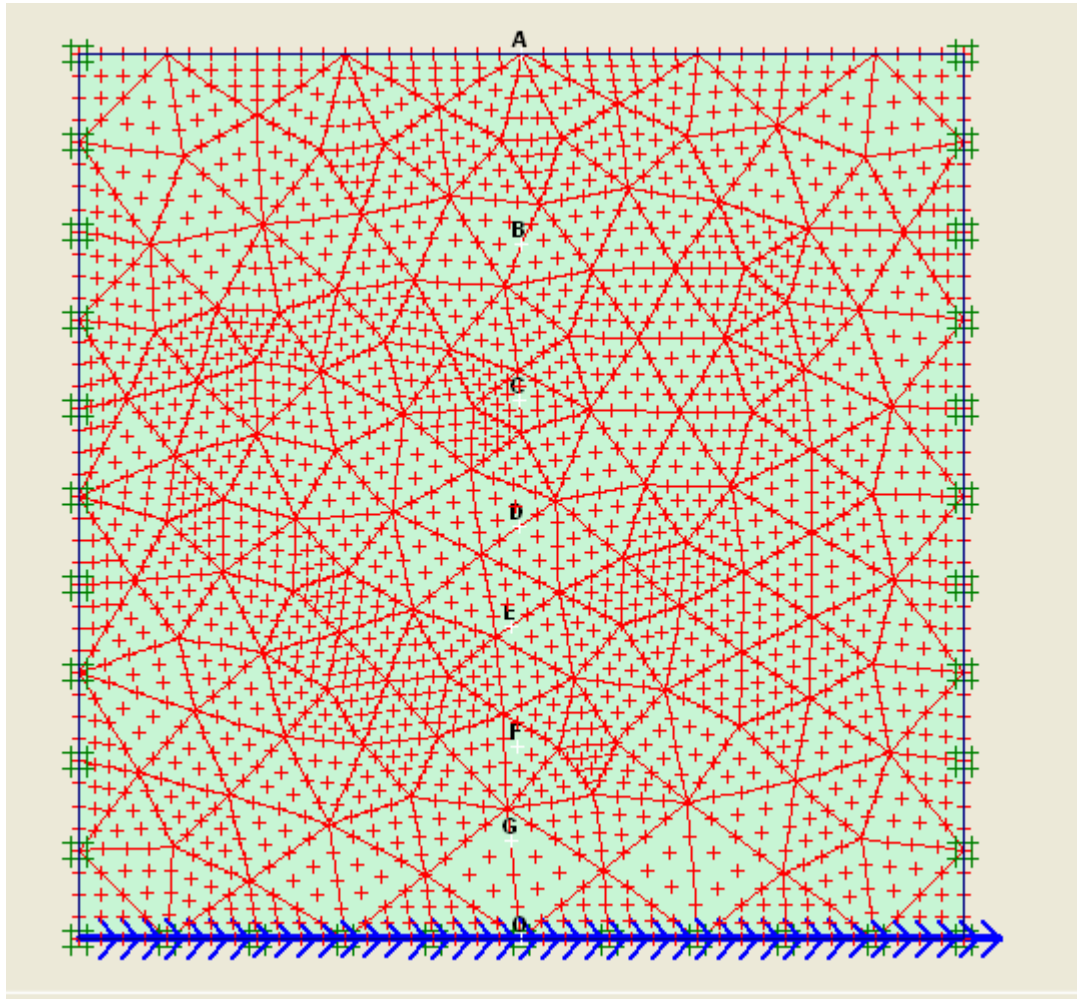


Undrained condition





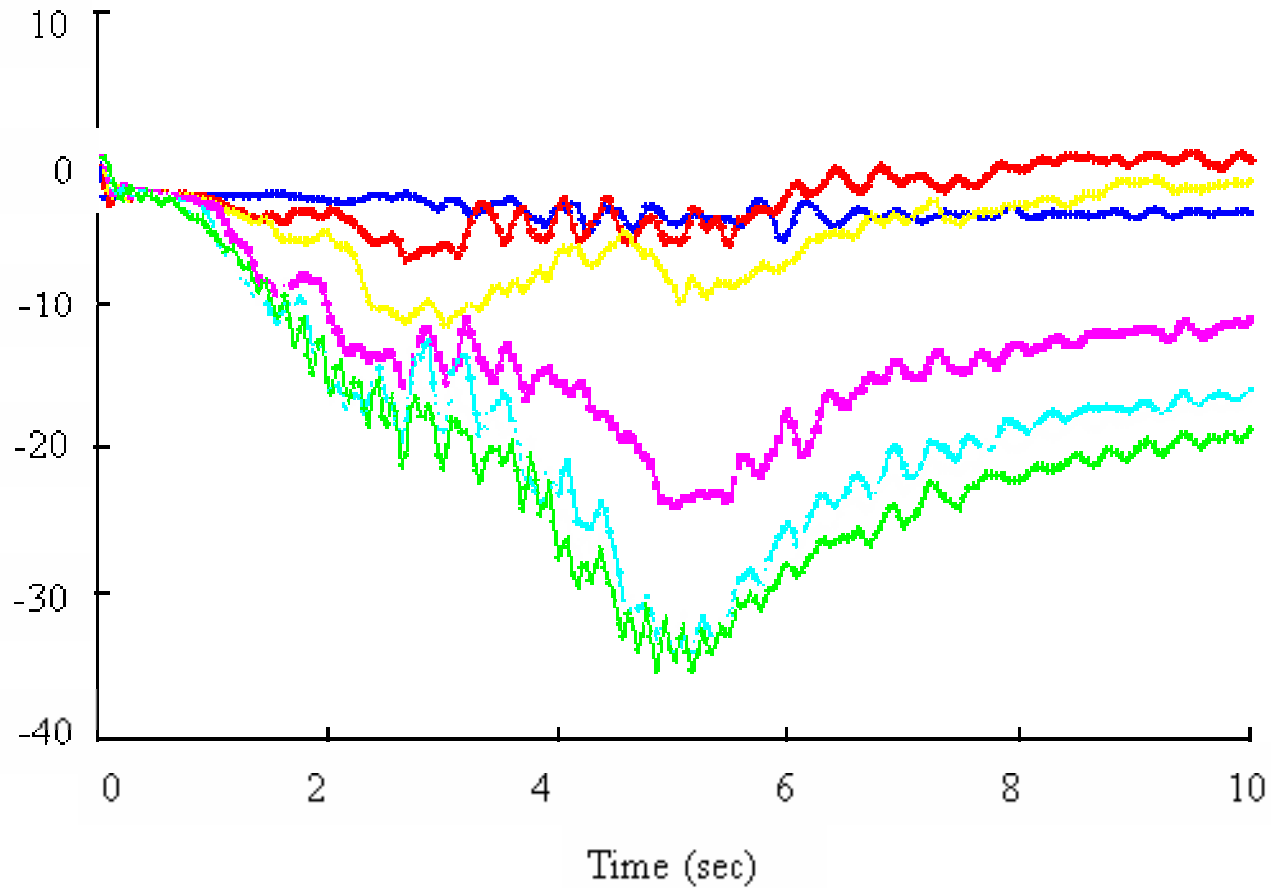
Input waves



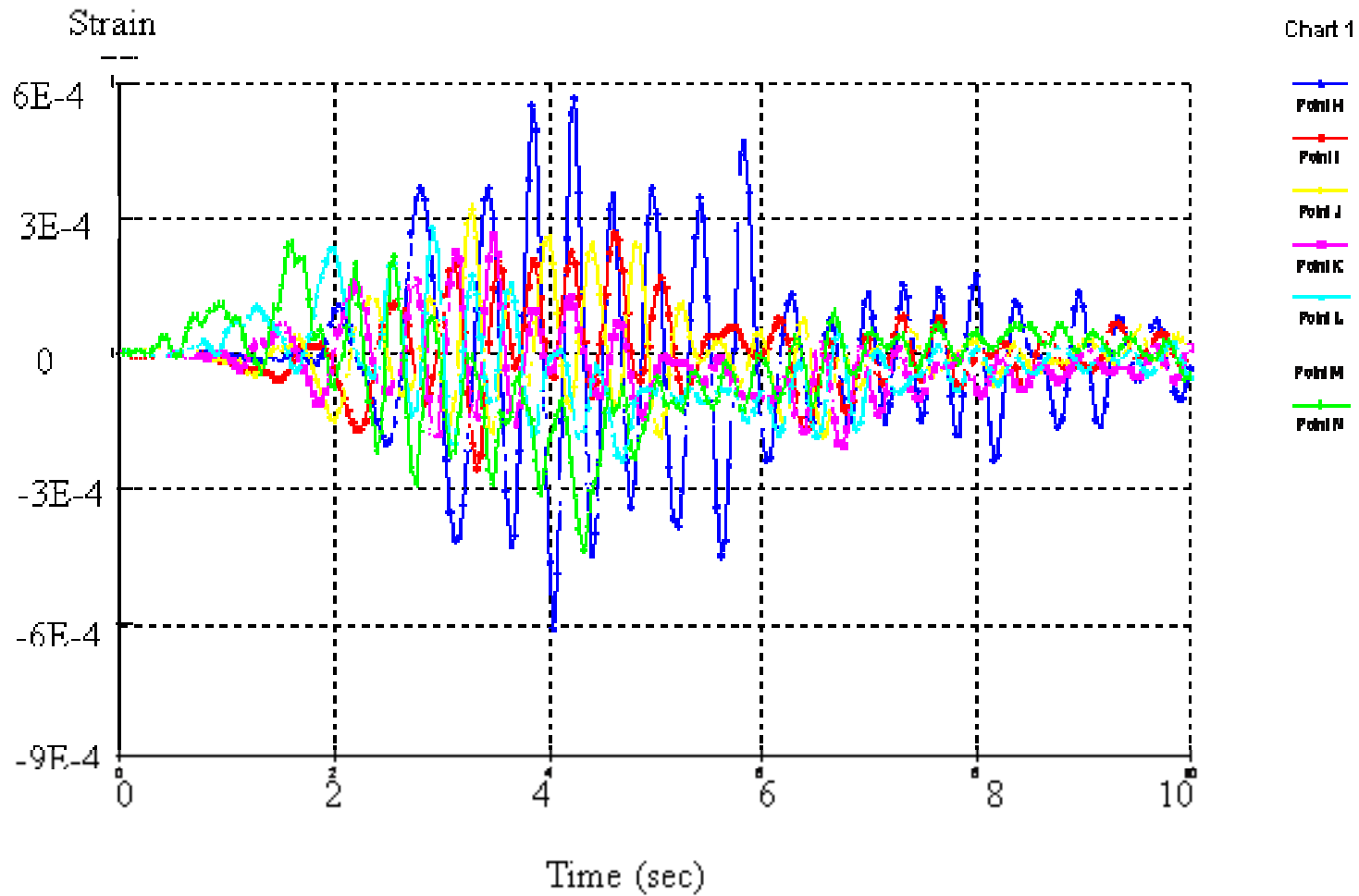
Configurations and output nodes

Dense Sand

Pore water pressure



Dense Sand



Dense Sand

Acceleration [m/s^2]

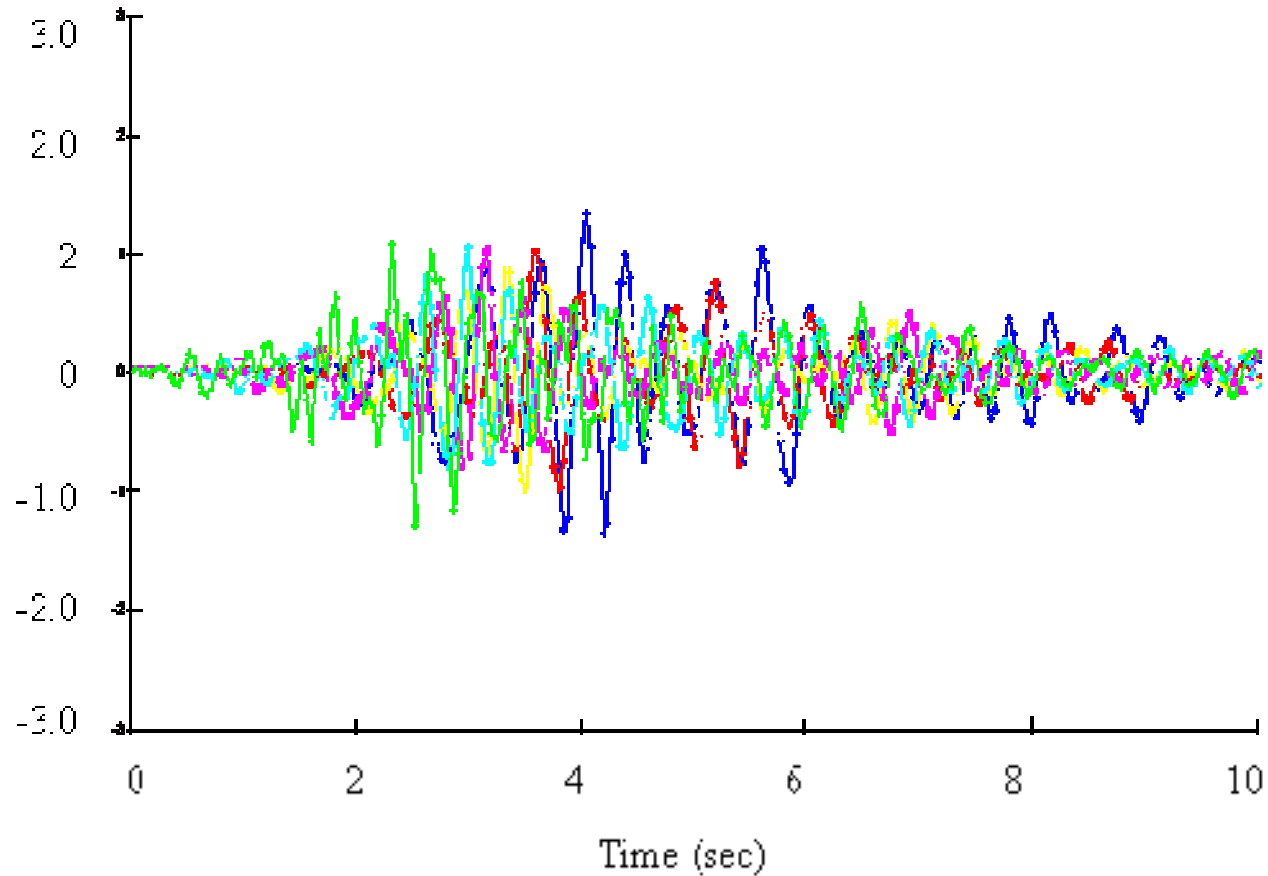


Chart 1

- Point A
- Point B
- Point C
- Point D
- Point E
- Point F
- Point G

Dense Sand

Acceleration [m/s^2]

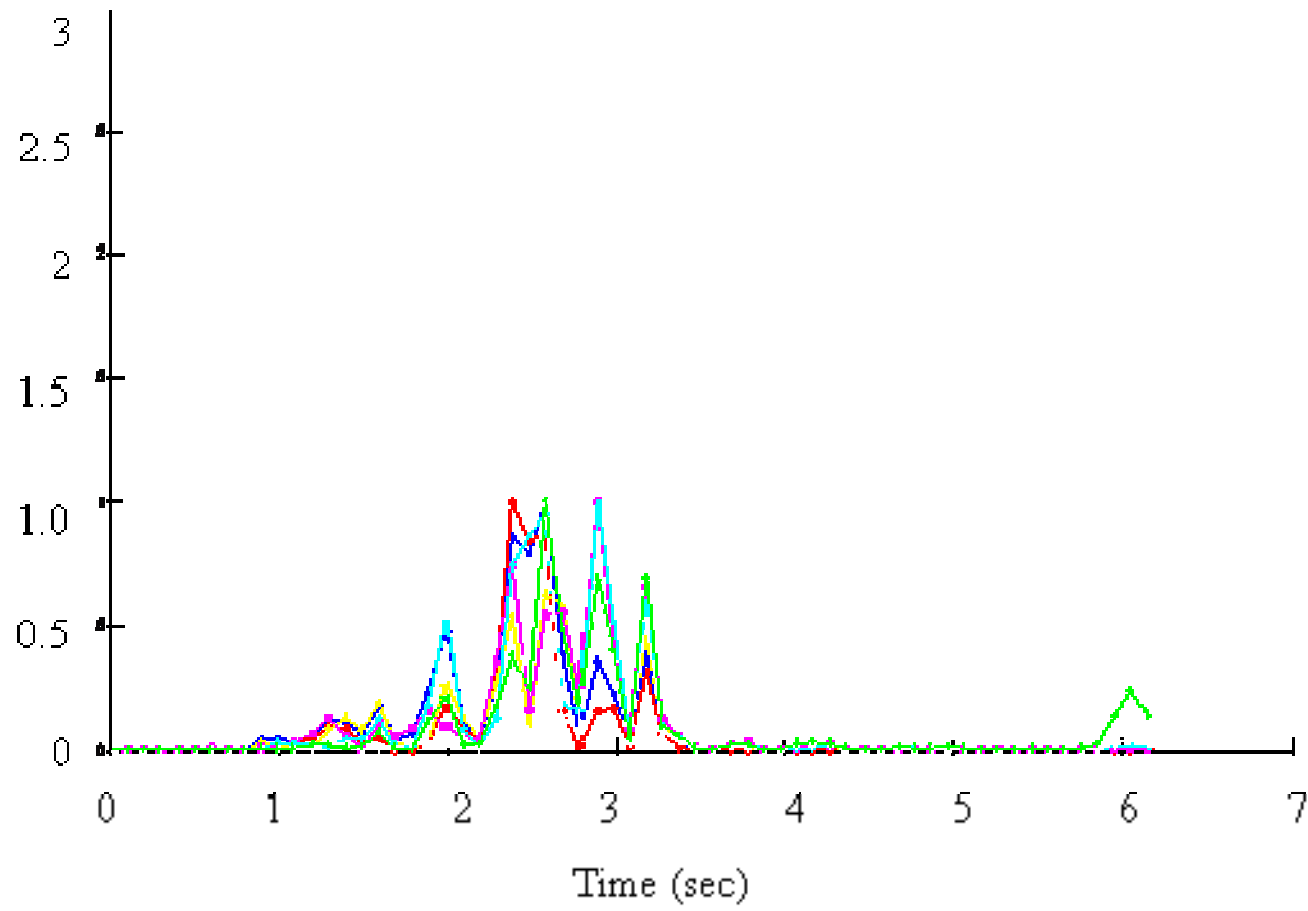
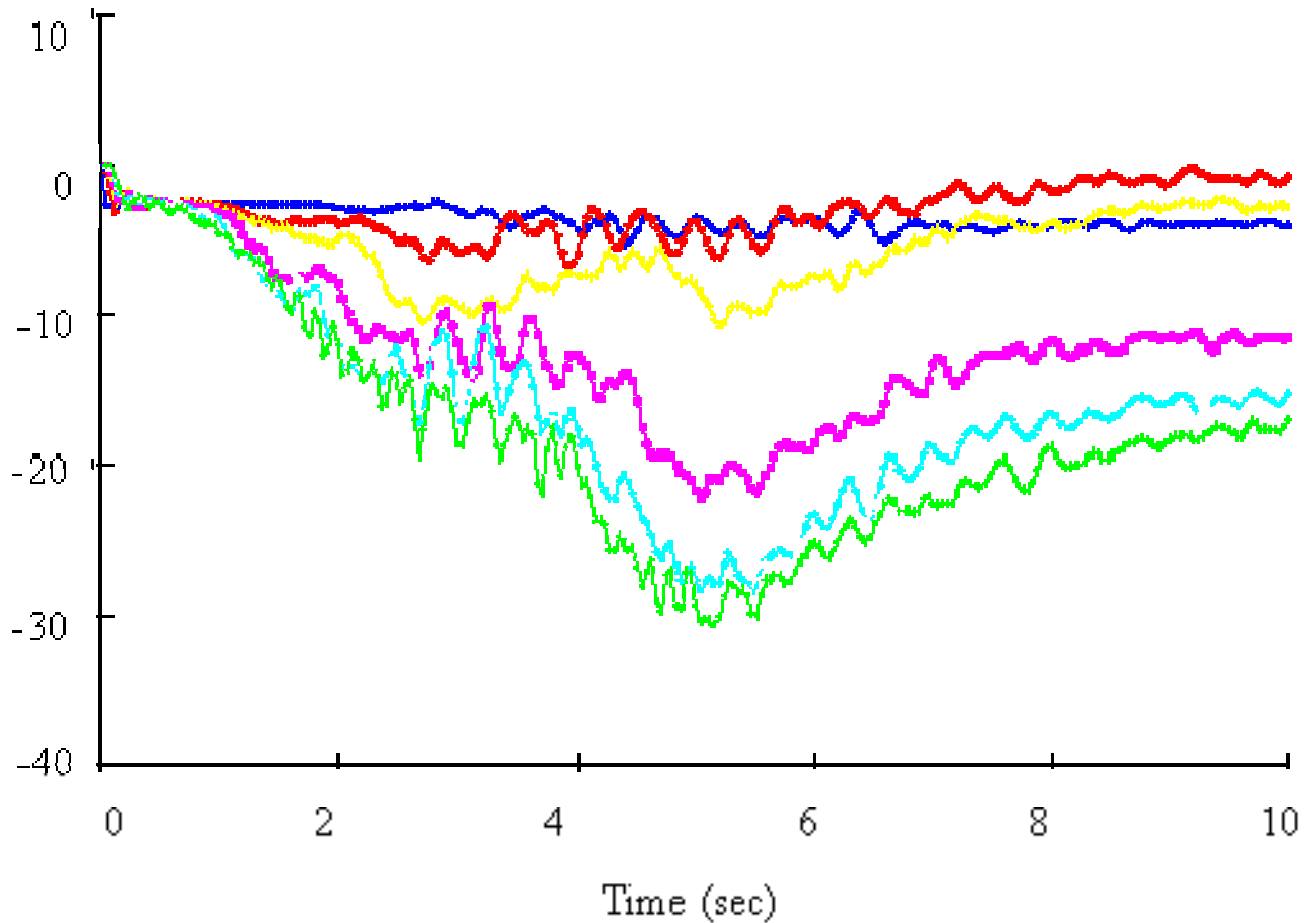


Chart 1

- Point A
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- Point F
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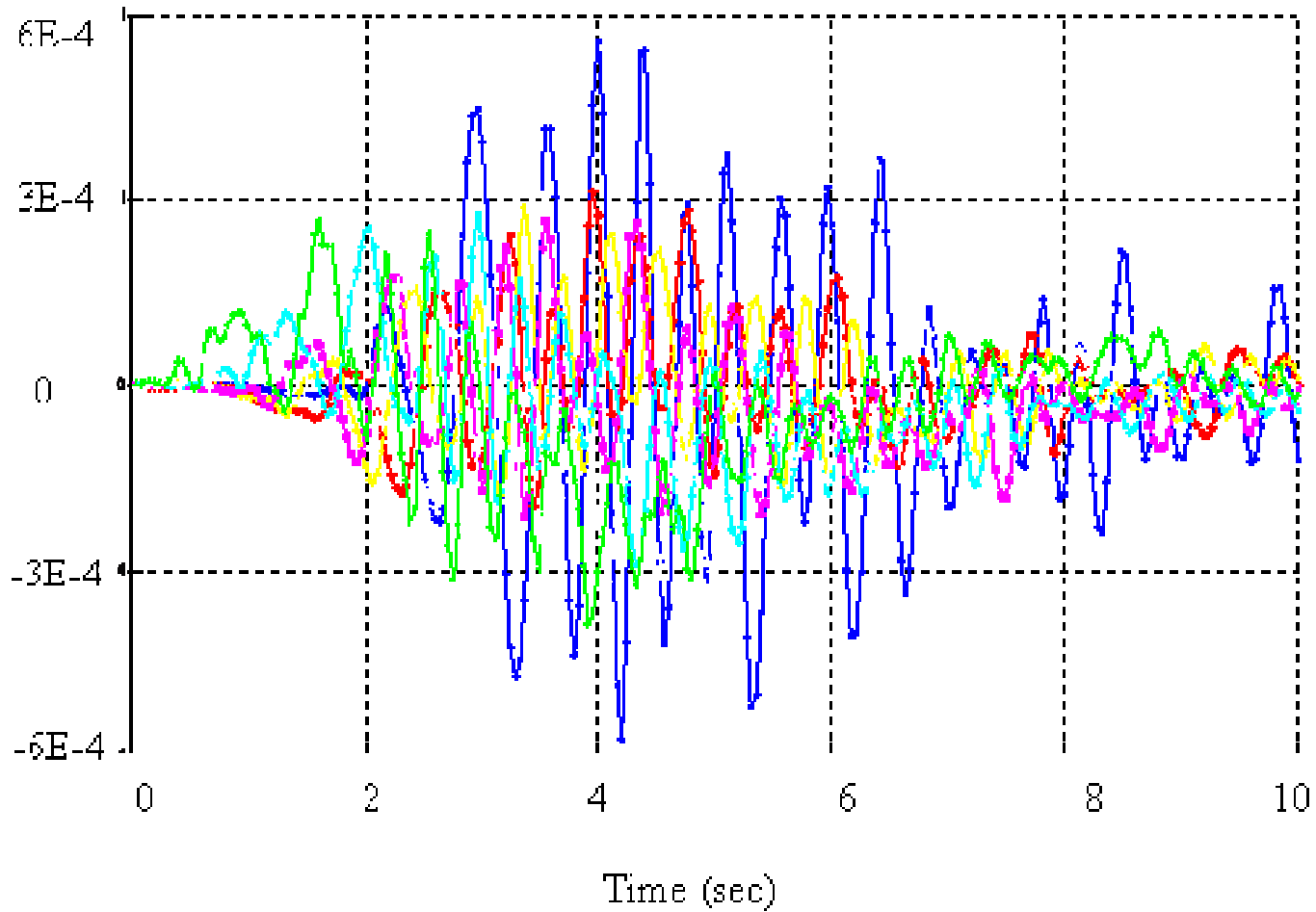
Medium Dense Sand

Pore water pressure



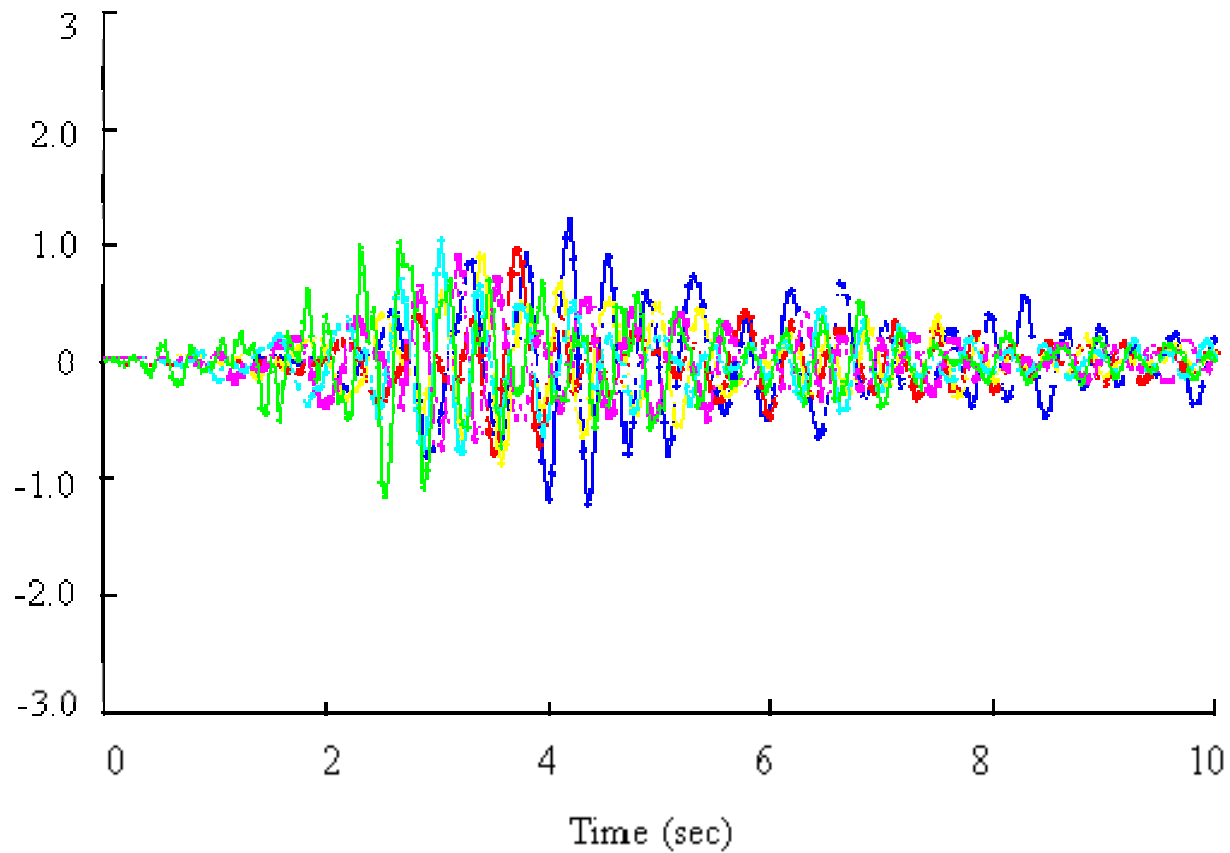
Medium Dense Sand

Shear Strain

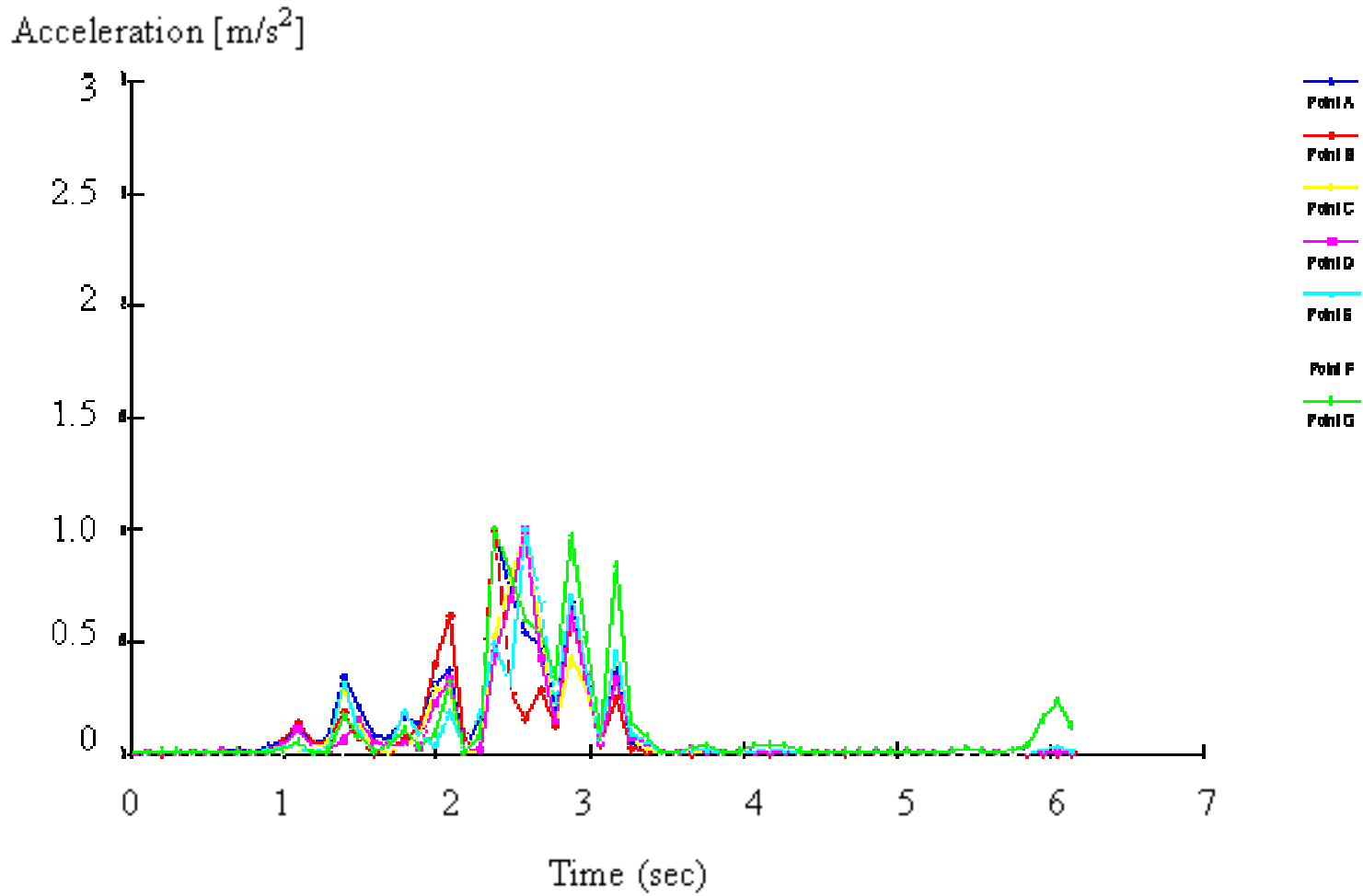


Medium Dense Sand

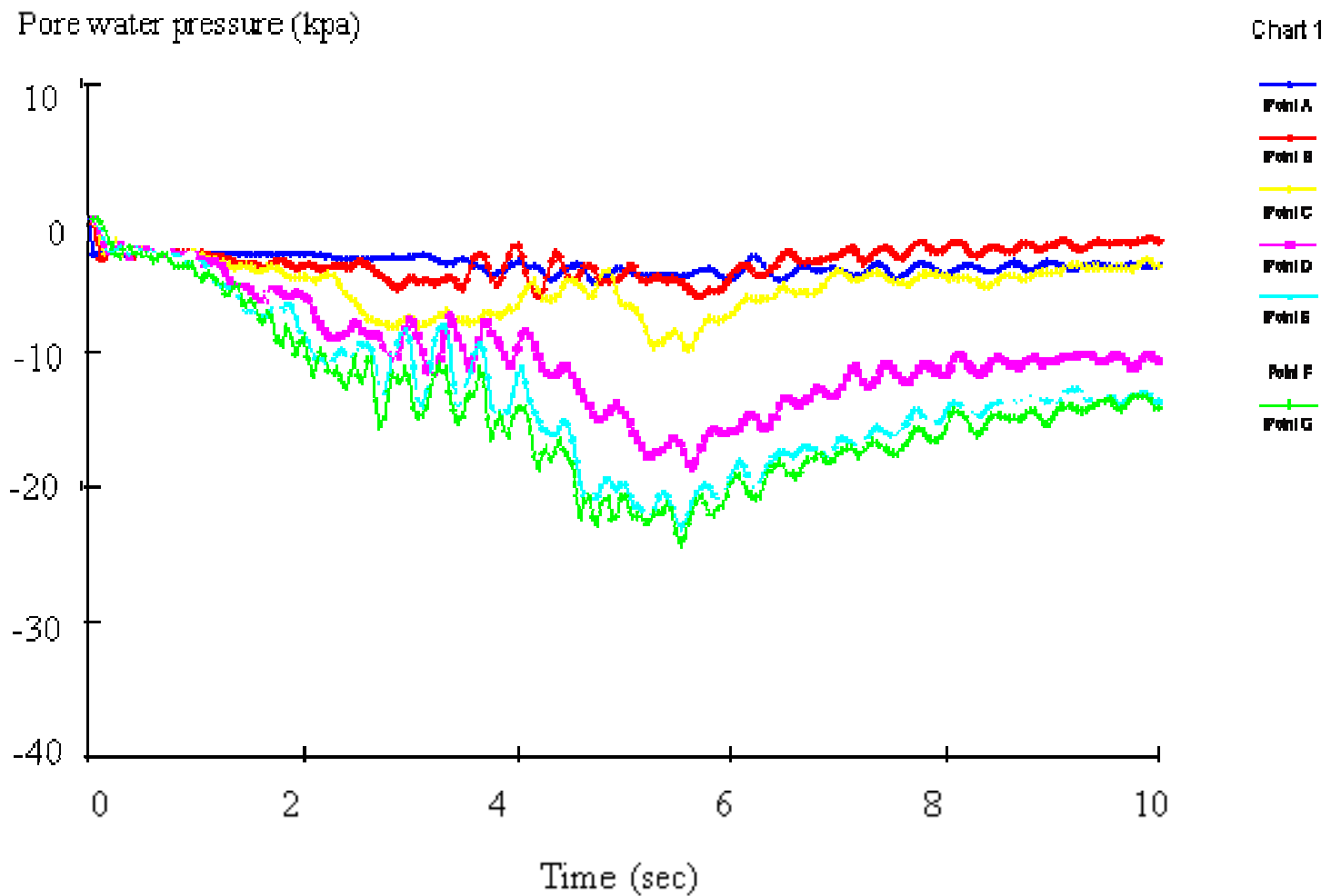
Acceleration [m/s^2]



Medium Dense Sand



Loose Sand



Shear Strain

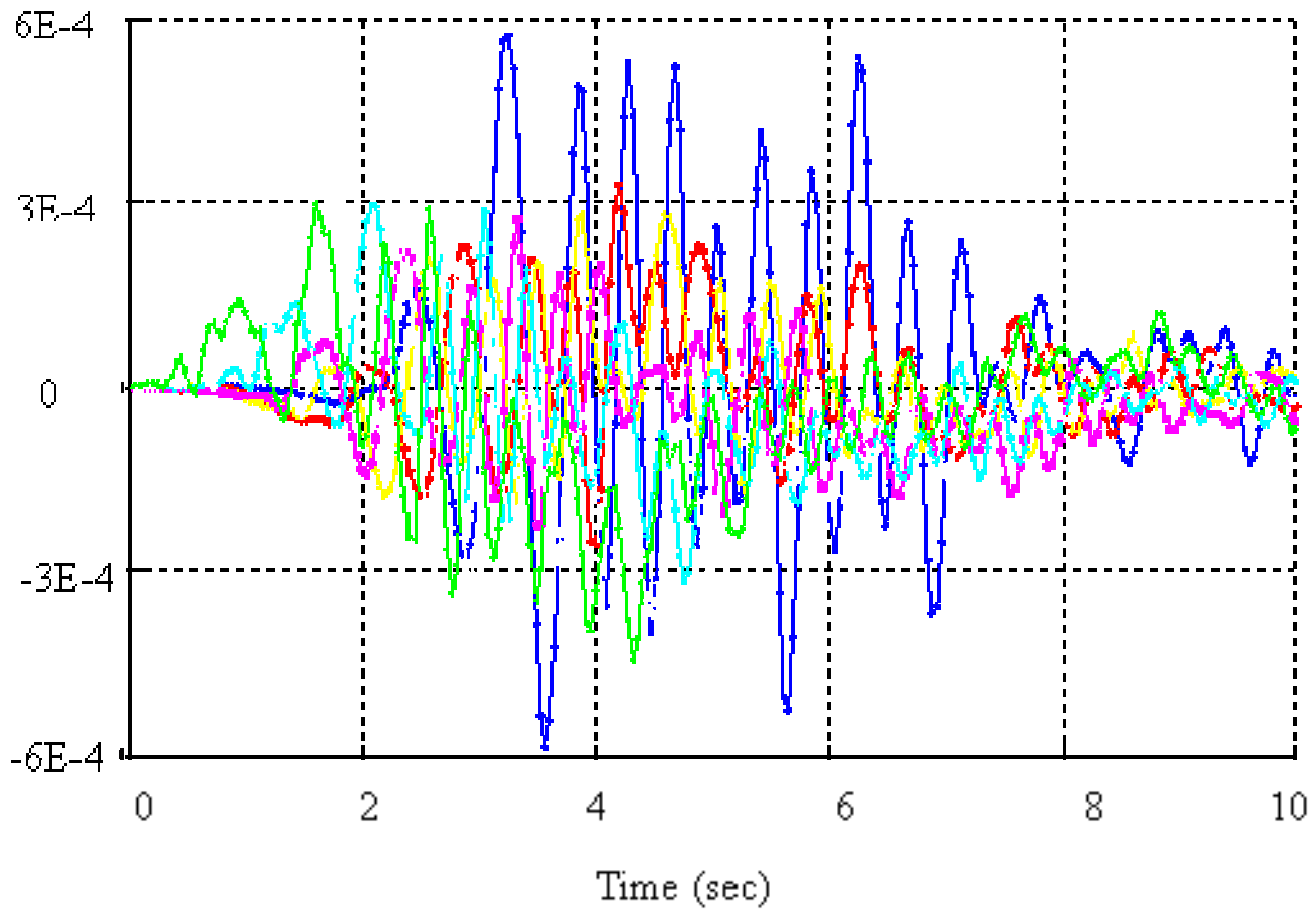
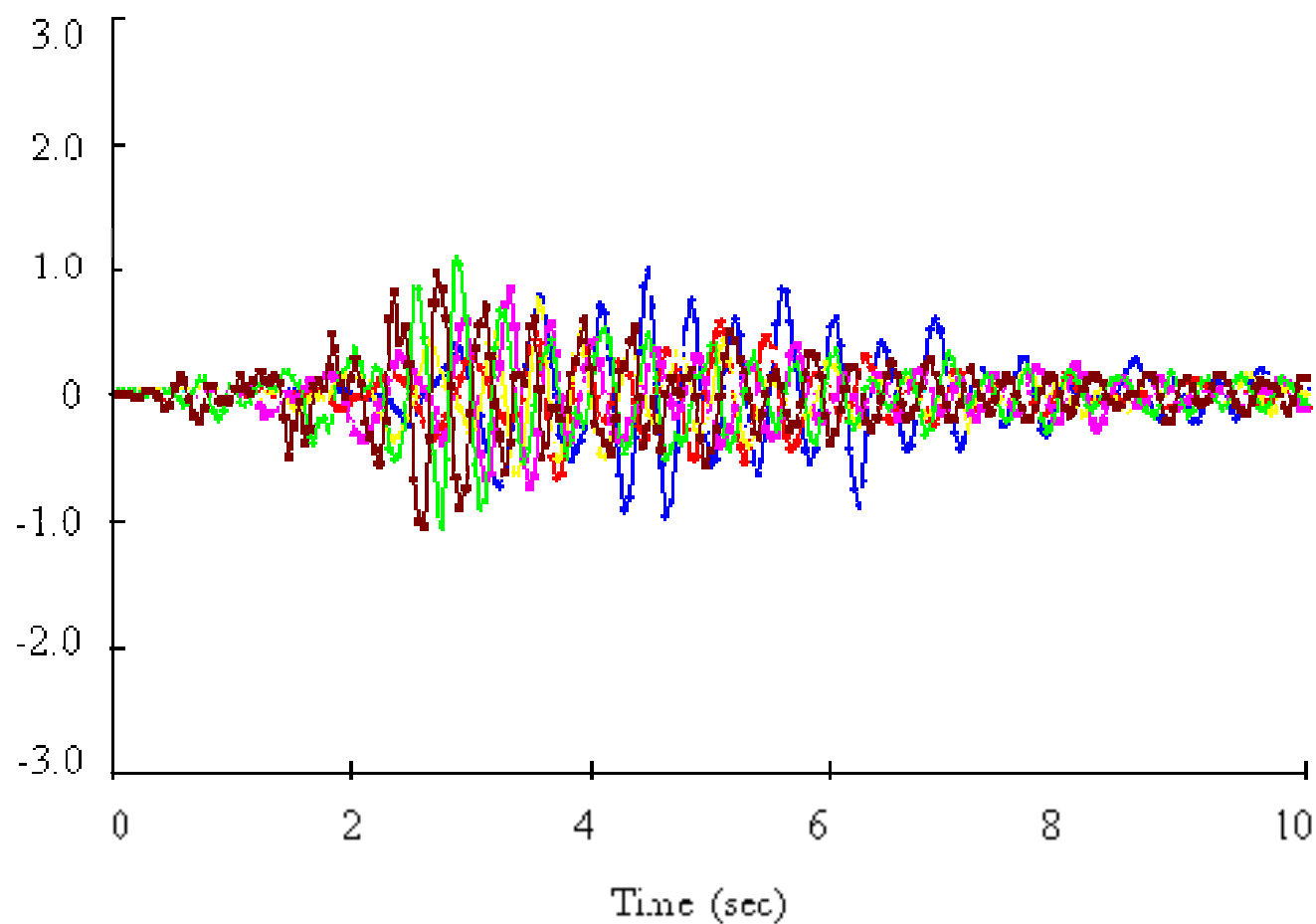


Chart 1

- Point H
- Point I
- Point J
- Point K
- Point L
- Point M
- Point N

Acceleration [$\text{m}\cdot\text{s}^{-2}$]

Chart 1



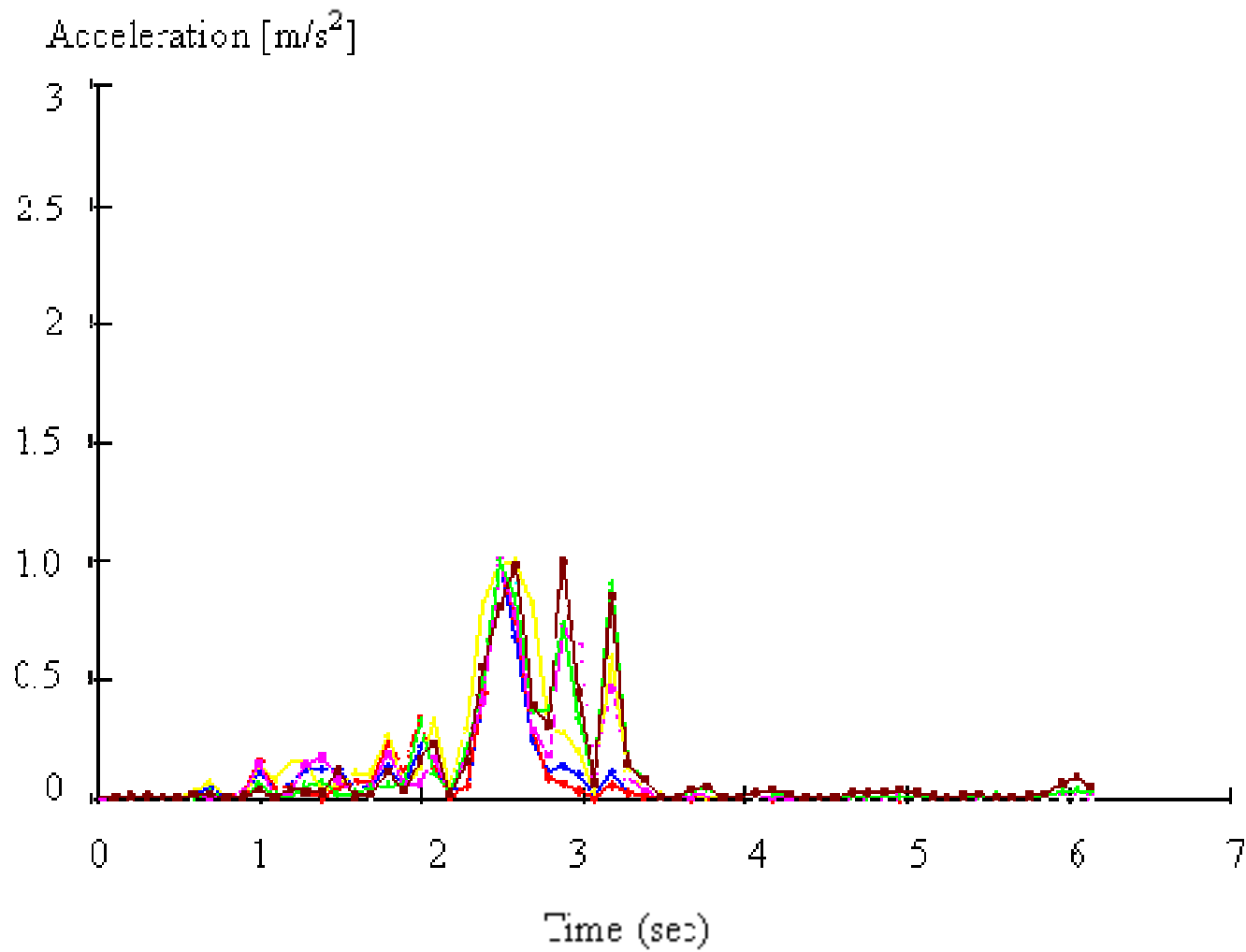
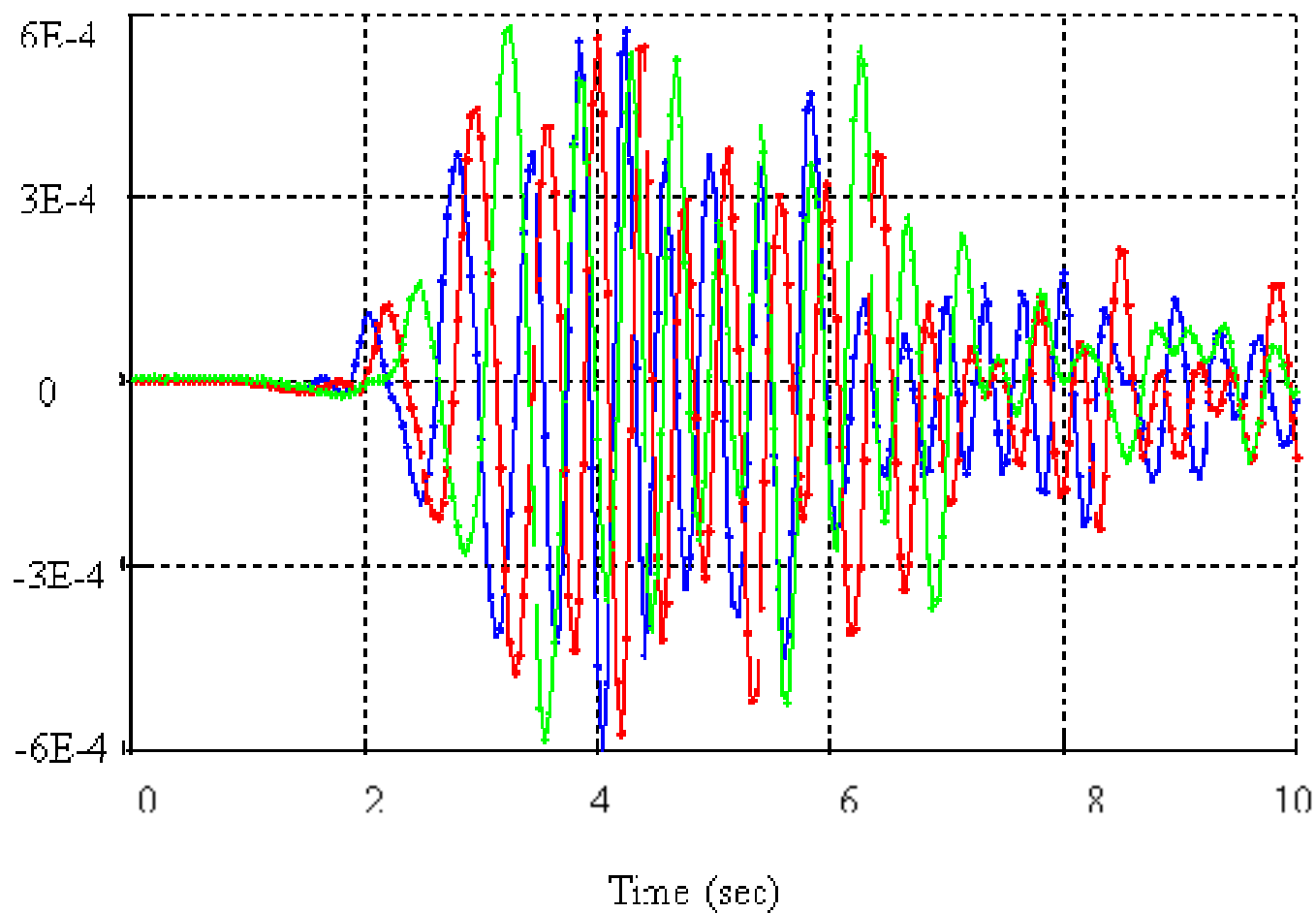


Chart 1

- Point A
- Point B
- Point C
- Point D
- Point E
- Point F
- Point G

Shear Strain

Chart 1



Conclusion (1/2)

- 1. The deep soil layers may be deformed upto $10E-4$ and it builds up the excess pore water pressure.
- 2. Excess pore water pressure in dense sandy soils increases more significant than in medium and loose sandy soils in the same deformation level at the beginning.
- 3. Excess pore water pressure of dense and medium sand reduces because of dilantancy.

Conclusion (2/2)

- 4. If a numerical simulation is conducted for deep sandy soils with the shear strain less than $10E-3$, elastic model can be applied for a quick and brief calculations.
- 5. The introduced numerical tool is capable of computing the seismic induced excess pore water pressure in the deep layers.