A Numerical Study of Effective Stress and Groundwater Level Changes in Poroelastic Aquifer under Dynamic Excitations

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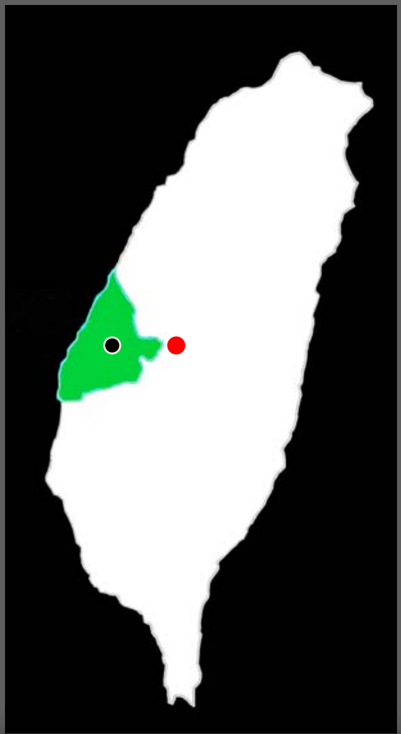
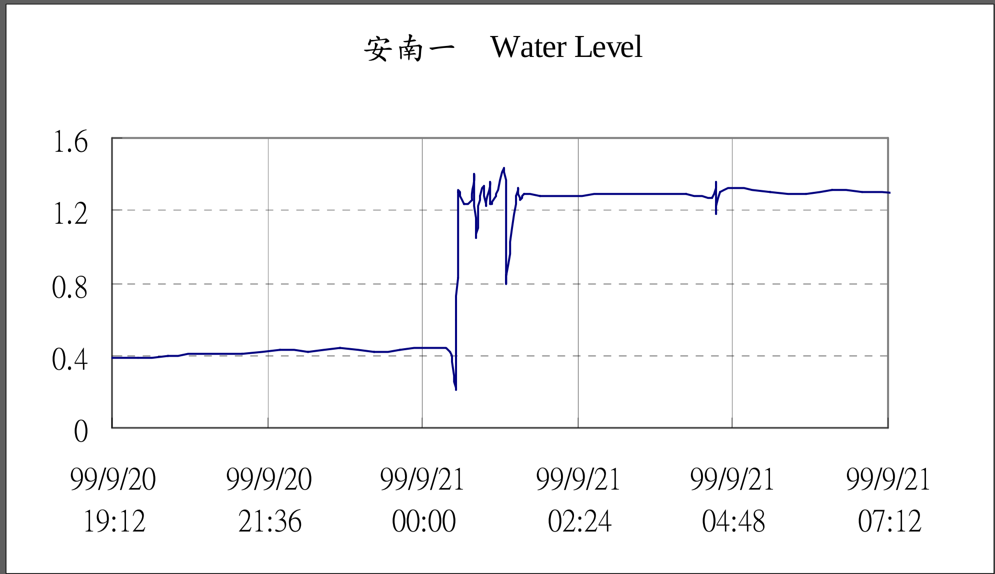
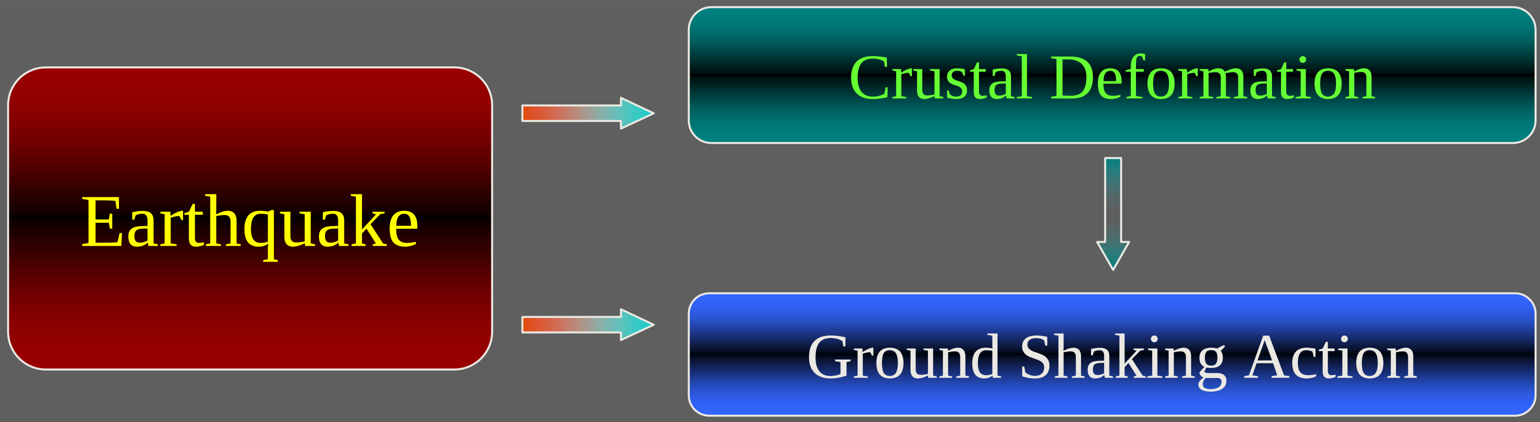
Ministry of Economic Affairs, Taiwan

Introduction





Mechanism


Earthquake
Crustal Deformation
Ground Shaking Action


Fluid
Solid

Objectives

- Modified dynamic poroelastic theory
- Numerical study
  - Sensitivity study
  - Effect of boundary condition
  - Stratum layer analysis
  - Case Study
    - Choshui River fan

The equations...

- Poroelasticity:
  - Biot(1941)
  - Rice & Cleary(1976)
  - Roeloffs(1996)
- Equations:
  - Law of Geometry
  - Law of Material Constitution
  - Law of Deformation and Flow
- Problem:
  - 2D plane strain

Biot's Classical Poroelasticity

- Basic Assumptions
  - Isotropy
  - Reversible process
  - Linear stress & strain constitution
  - Infinitesimal deformation
  - Incompressible fluid
  - Darcy's flow law

Biot’s original equations

$$G\nabla^2 u_x + \frac{G}{1-2\upsilon}\frac{\partial \varepsilon}{\partial x} - \alpha\frac{\partial p}{\partial x} = \rho\frac{\partial^2 u_x}{\partial t^2}$$

$$G\nabla^2 u_y + \frac{G}{1-2\upsilon}\frac{\partial \varepsilon}{\partial y} - \alpha\frac{\partial p}{\partial y} = \rho\frac{\partial^2 u_y}{\partial t^2}$$

$$G\nabla^2 u_z + \frac{G}{1-2\upsilon}\frac{\partial \varepsilon}{\partial z} - \alpha\frac{\partial p}{\partial z} = \rho\frac{\partial^2 u_z}{\partial t^2}$$

$$\frac{k}{\gamma_w}\nabla^2 p = \alpha\frac{\partial \varepsilon}{\partial t} + \frac{1}{Q}\frac{\partial p}{\partial t}$$

Law of infinitesimal deformation

The total strain - displacement relations in plane strain :

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} = 0 \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{bmatrix} \begin{Bmatrix} u_x \\ u_y \\ u_z = \text{constant} \end{Bmatrix}$$

HILE porous materials

For homogeneous isotropic linear elastic porous materials:

The total stress - strain relations in plane strain:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 \\ \nu & 1-\nu & \nu & 0 \\ \nu & \nu & 1-\nu & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} = 0 \\ \gamma_{xy} \end{Bmatrix}$$

The effective stress concept:

$$\begin{Bmatrix} \sigma^e_{xx} \\ \sigma^e_{yy} \\ \sigma^e_{zz} \\ \tau_{xy} \end{Bmatrix} = \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \end{Bmatrix} + \alpha \begin{Bmatrix} p \\ p \\ p \\ 0 \end{Bmatrix}$$

where + stands for tension.

$$\alpha = \frac{3(\nu_u - \nu)}{B(1+\nu_u)}$$

Governing law of deformation

The dynamic stress equations:

$$\frac{\partial \sigma^e{}_{xx}}{\partial x} + \frac{\partial \tau^e{}_{xy}}{\partial y} = \rho \frac{\partial^2 u_x}{\partial t^2} + \zeta \frac{\partial u_x}{\partial t}$$

$$\frac{\partial \tau^e{}_{yx}}{\partial x} + \frac{\partial \sigma^e{}_{yy}}{\partial y} = \rho \frac{\partial^2 u_y}{\partial t^2} + \zeta \frac{\partial u_y}{\partial t}$$

Damping

Inertia

where

$\sigma^e{}_{xx}, \sigma^e{}_{yy}, \tau^e{}_{xy}$ = effective stress components (Pa = N/m^2),

(u_x, u_y) = displacement vector (m),

ρ = mass density (kg/m^3),

ζ = damping coefficient (kg/m^{3}s),

Governing law of flow

The flow equation :

$$\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_{yy}\frac{\partial p}{\partial y}\right)=S_o\frac{\partial p}{\partial t}+\chi\frac{\partial \varepsilon}{\partial t}$$

where

p = pore pressure (Pa = N/m^2),

$\varepsilon = \frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}+\frac{\partial u_z}{\partial z}$ = volumetric strain,

K_{xx}, K_{yy} = hydraulic conductivity in x and y directions(m/s),

S_o = storage coefficient(1/m),

χ = dilation amplifying coefficient(Pa/m).

can be related to strain efficiency

In classical approach, χ should be related to α, but we treat χ independently.

Formulation

$$V_x = \frac{d(u_x)}{dt}$$

$$V_z = \frac{d(u_z)}{dt}$$

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} = \rho \frac{\partial^2 u_x}{\partial t^2} + \zeta V_x$$

$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} = \rho \frac{\partial^2 u_z}{\partial t^2} + \zeta V_z$$

$$k\nabla^2 h = \frac{n\beta_f}{\gamma_w}\frac{\partial h}{\partial t} + \chi \frac{\partial \varepsilon}{\partial t}$$

$$\frac{V_x}{gT_0} = \frac{\partial\left(\frac{u_x}{gT_0^{\,2}}\right)}{\partial \bar{t}}$$

$$\frac{V_z}{gT_0} = \frac{\partial\left(\frac{u_z}{gT_0^{\,2}}\right)}{\partial \bar{t}}$$

$$\frac{\partial\left(\frac{\sigma^e{}_{xx}}{\rho g X_0}\right)}{\partial \bar{x}} + \frac{\partial\left(\frac{\sigma_{zx}}{\rho g Z_0}\right)}{\partial \bar{z}} = \frac{\partial\left(\frac{V_x}{gT_0}\right)}{\partial \bar{t}} + \zeta \frac{V_x}{\rho g}$$

$$\frac{\partial\left(\frac{\sigma_{xz}}{\rho g X_0}\right)}{\partial \bar{x}} + \frac{\partial\left(\frac{\sigma^e{}_{zz}}{\rho g Z_0}\right)}{\partial \bar{z}} = \frac{\partial\left(\frac{V_z}{gT_0}\right)}{\partial \bar{t}} + \zeta \frac{V_z}{\rho g}$$

$$\frac{\partial(\bar{h})}{\partial \bar{x}^2} + A\frac{\partial(\bar{h})}{\partial \bar{z}^2} = B\frac{\partial(\bar{h})}{\partial \bar{t}} + C\frac{\partial(\varepsilon)}{\partial \bar{t}}$$

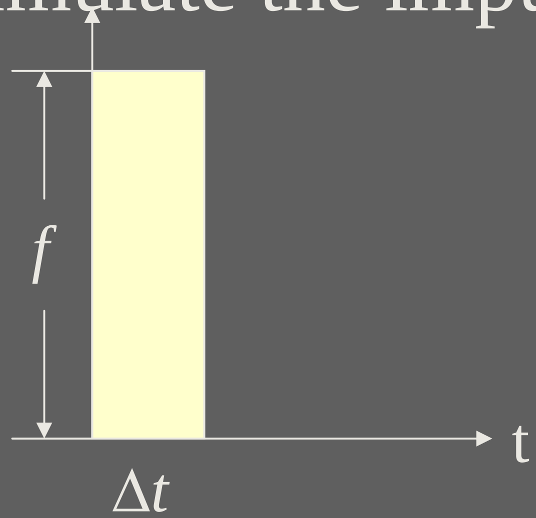
ζ ：Damping coefficient

χ ：Volumetric strain amplifying coefficient

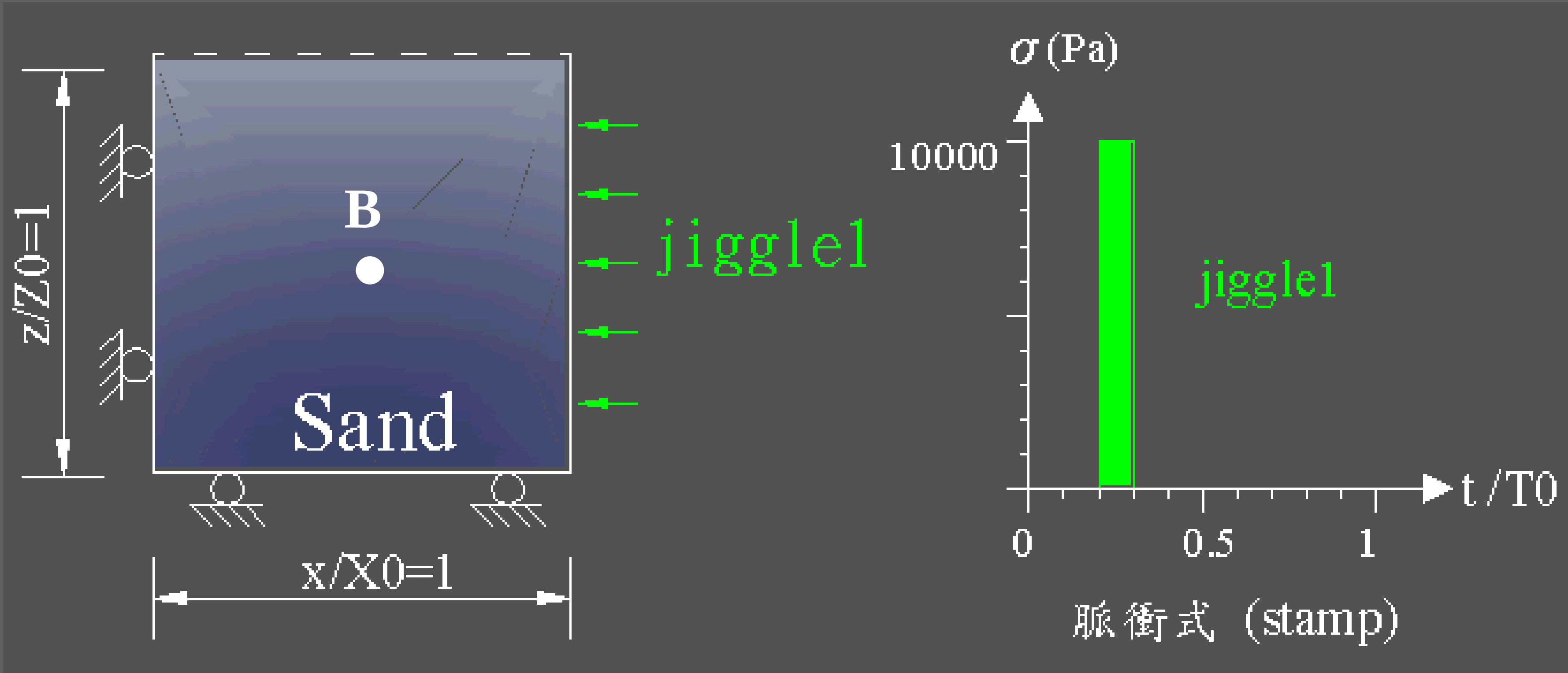
$$A = \frac{X_0^{\,2}}{k_{xx}}\frac{k_{zz}}{Z_0^{\,2}},\quad B = \frac{X_0^{\,2}}{k_{xx}}\frac{\gamma_w}{QT_0},\quad C = \frac{X_0^{\,2}}{k_{xx}}\frac{\chi}{T_0 H_0}$$

Excitations

- A “stamp like” function is used to simulate the jiggle driving force.
- The interval, Δt, is chosen to be a small quantity to simulate the impulse-type force.



The Numerical Model



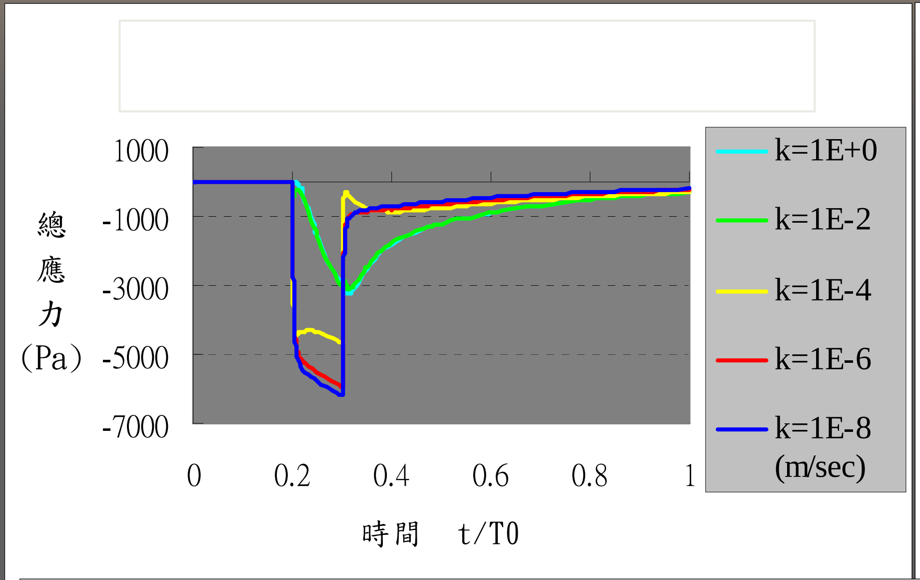
Basic Input Parameters

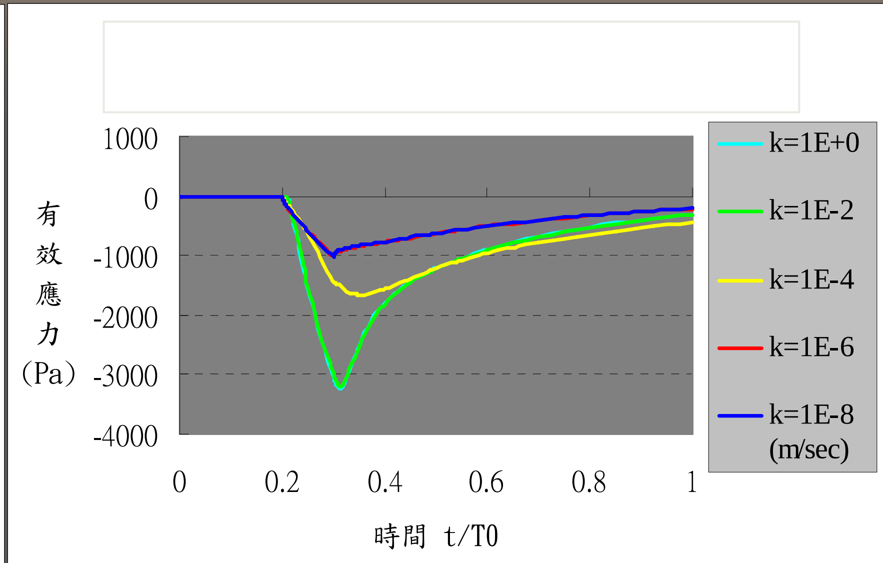
	Symbol	Sand	Clay	Unit
	υ : Poisson ratio	0.25	0.3	—
	E : Young's coefficient	1E+8	1E+7	N/m^2(Pa)
	ρ : Density	2100	1870	kg/m^3
	n : Porosity	0.375	0.55	—
	γ_w : Unit weight of water	9810	9810	N/m^3
	K : Hydraulic conductivity	1E-4	1E-6	m/sec
	β_f : Fluid compressibility	4.4E-10	4.4E-10	m^2/N (Pa^{-1})
	ζ : Damping coefficient	1E+10		kg/m^3.s
	χ : Volumetric strain amplifying coefficient	1		—

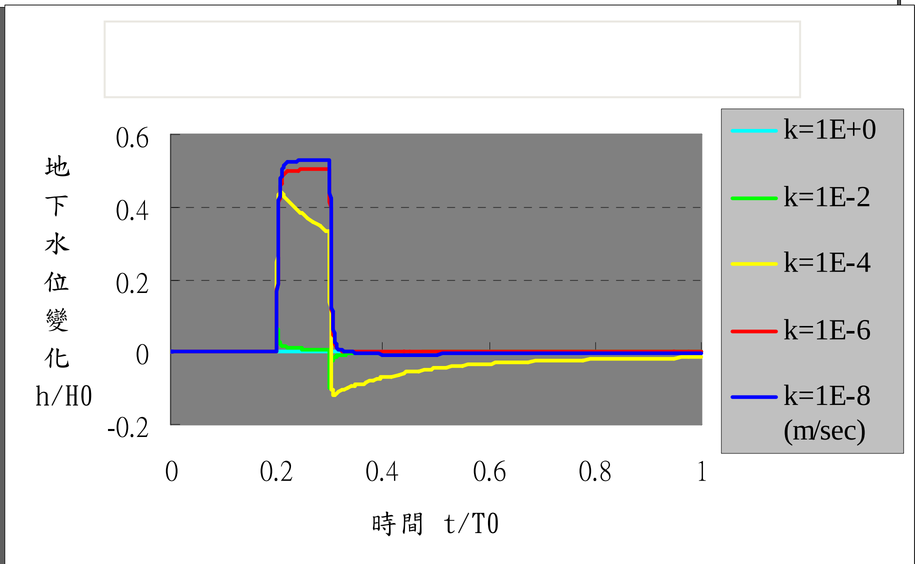
Sensitivity Study

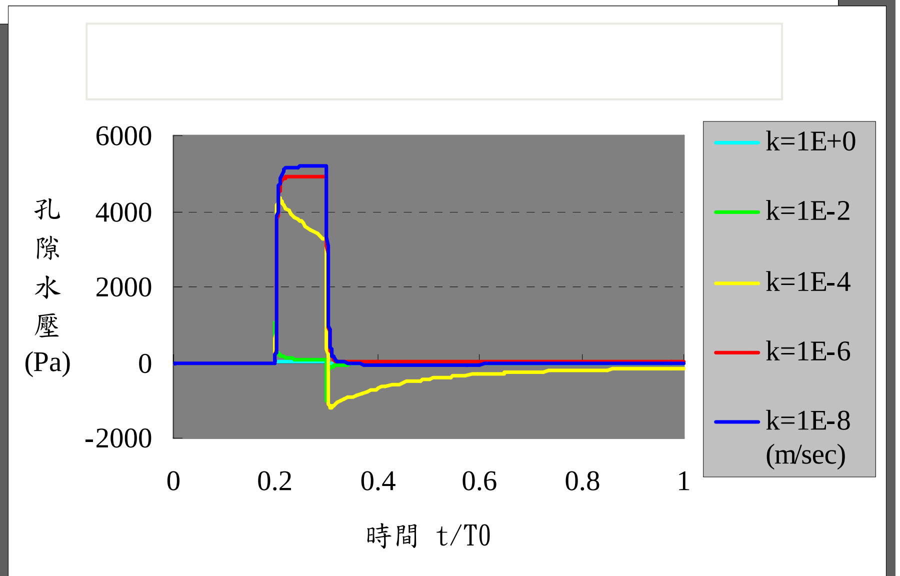
- **Hydraulic conductivity**
- **Young’s modulus**
- **Strain amplification coefficient**
- **Damping coefficient**
- **Amplitude of Excitations**

Hydraulic Conductivity

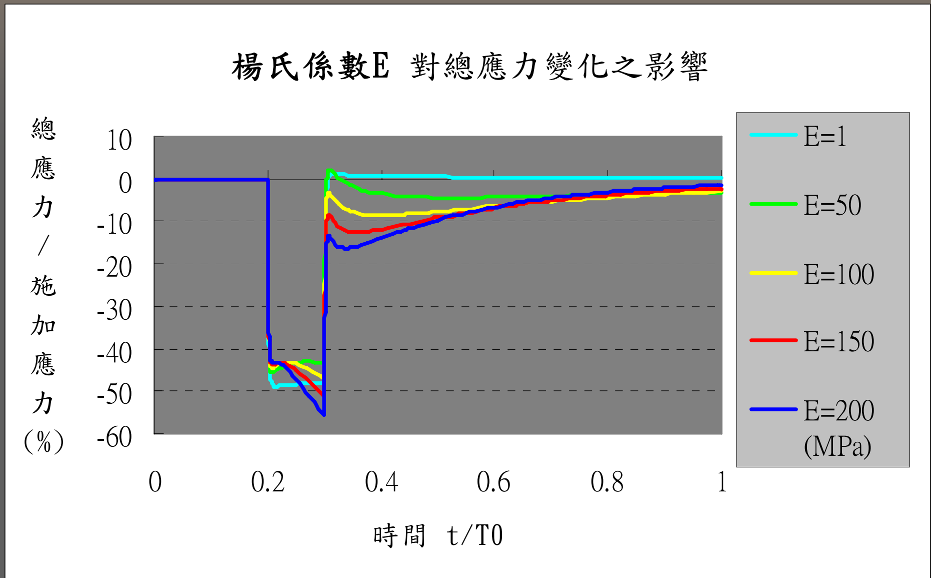


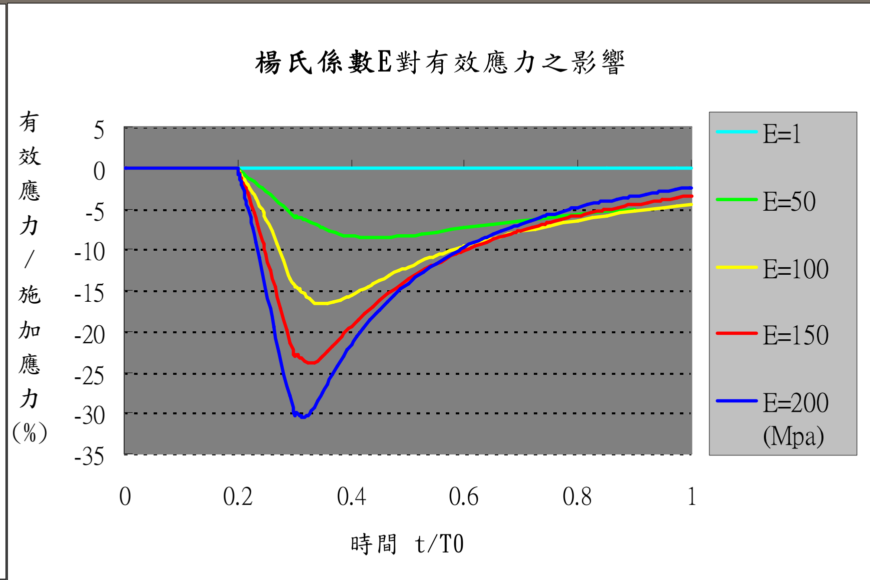


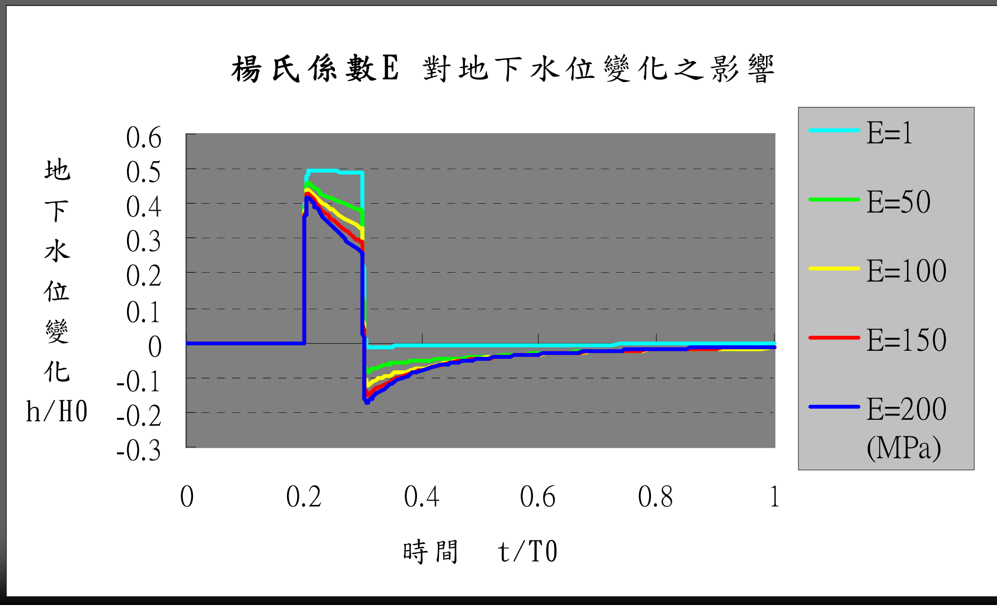




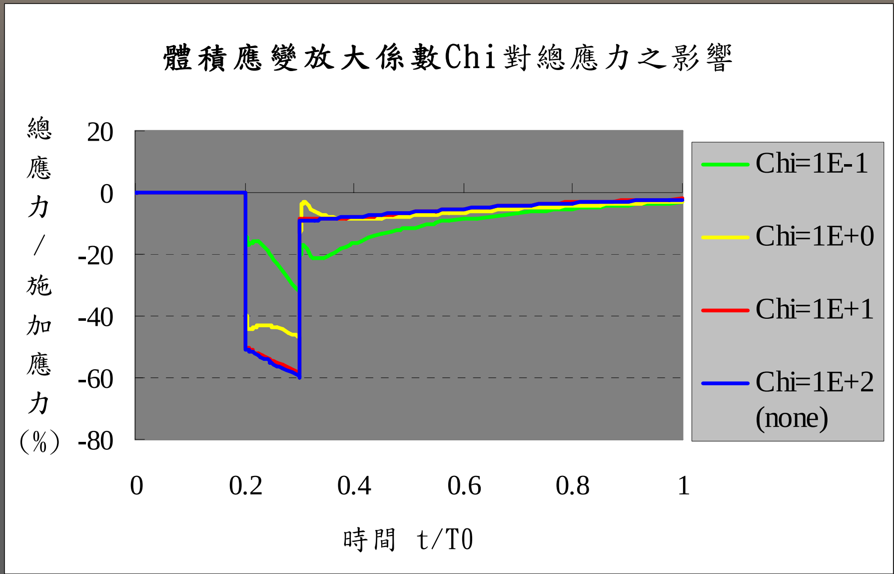
Young's modulus

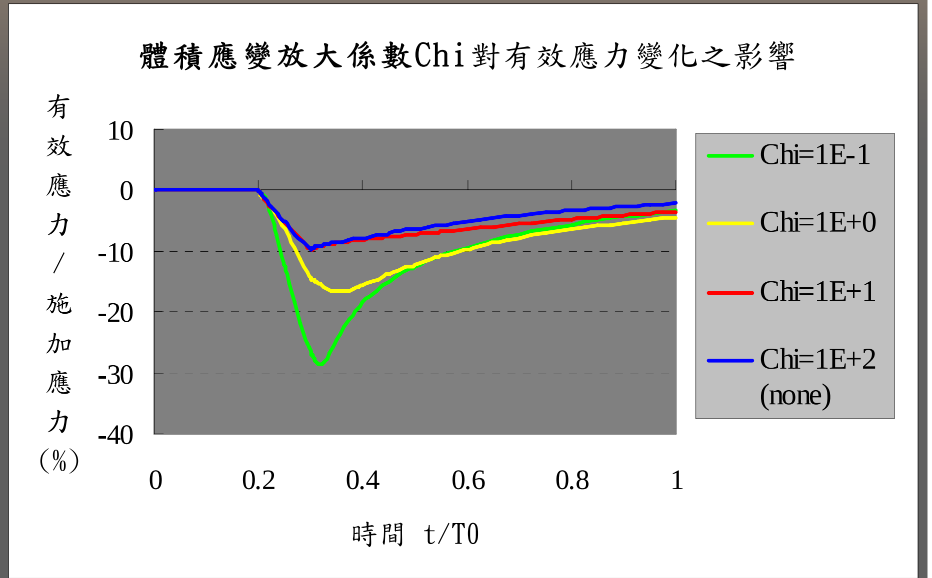


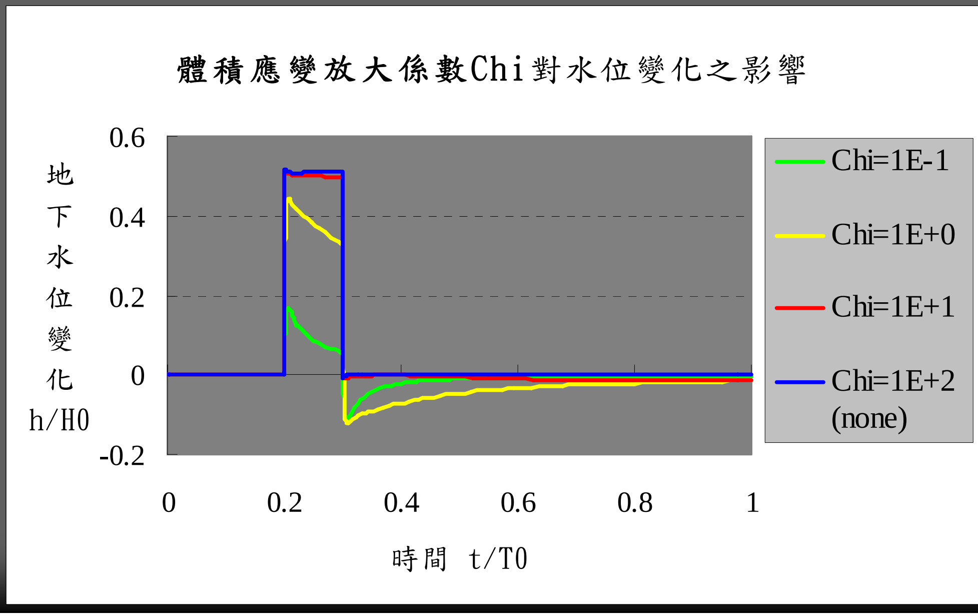




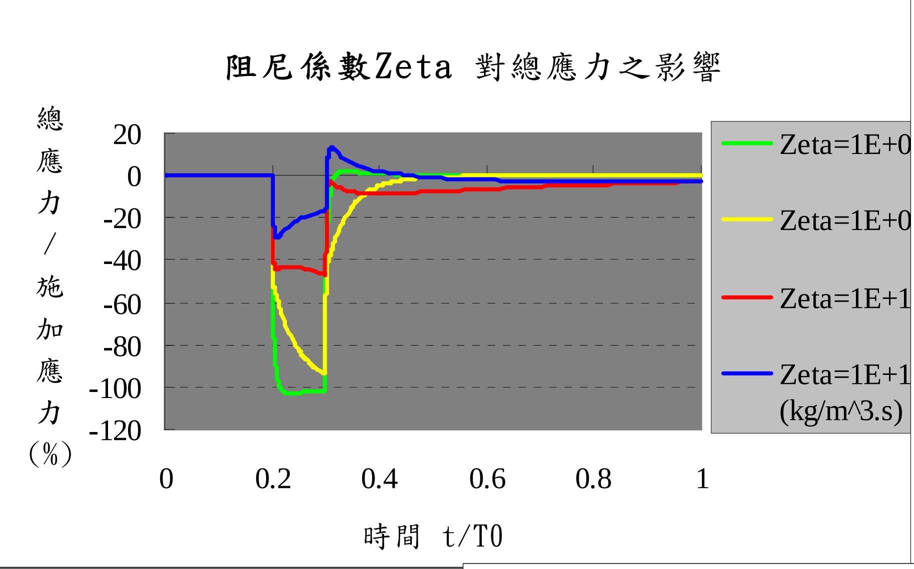
Strain amplification coefficient

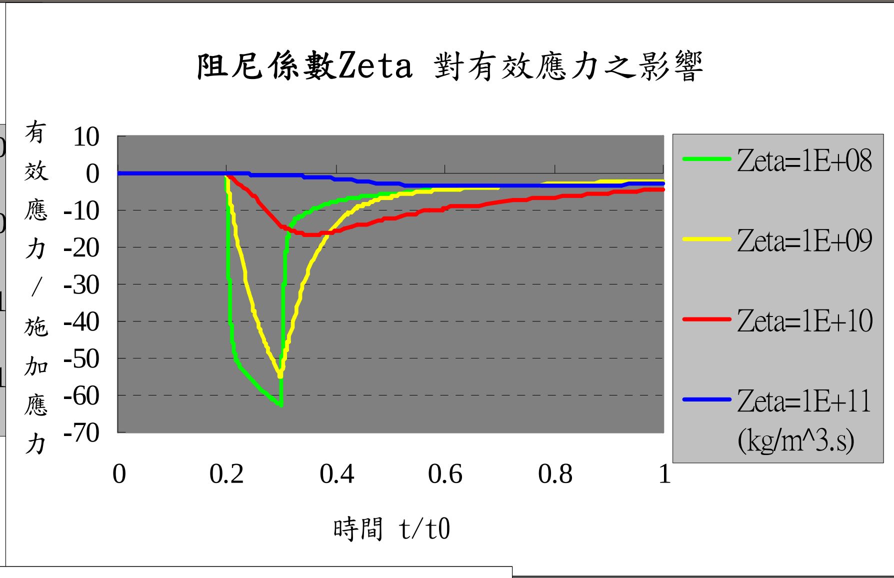


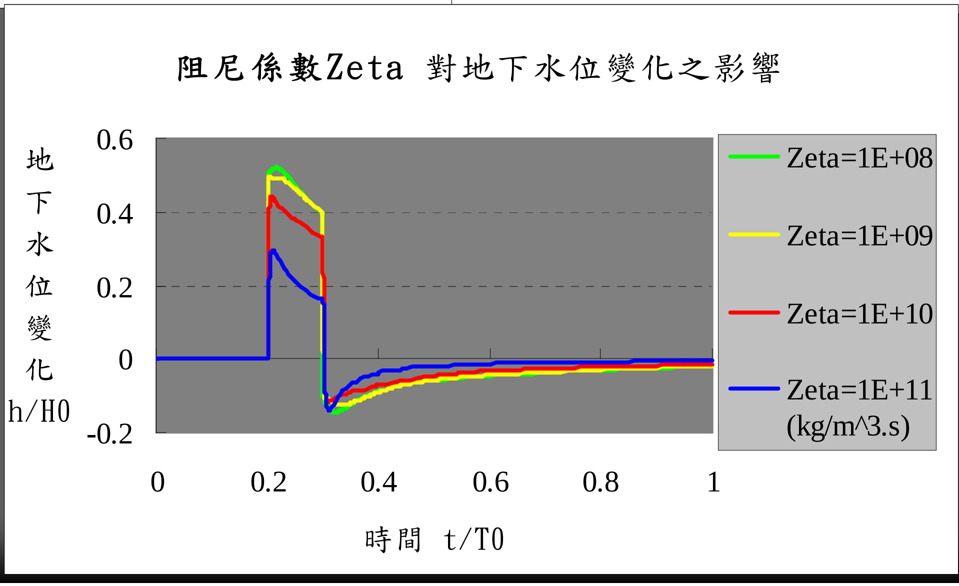




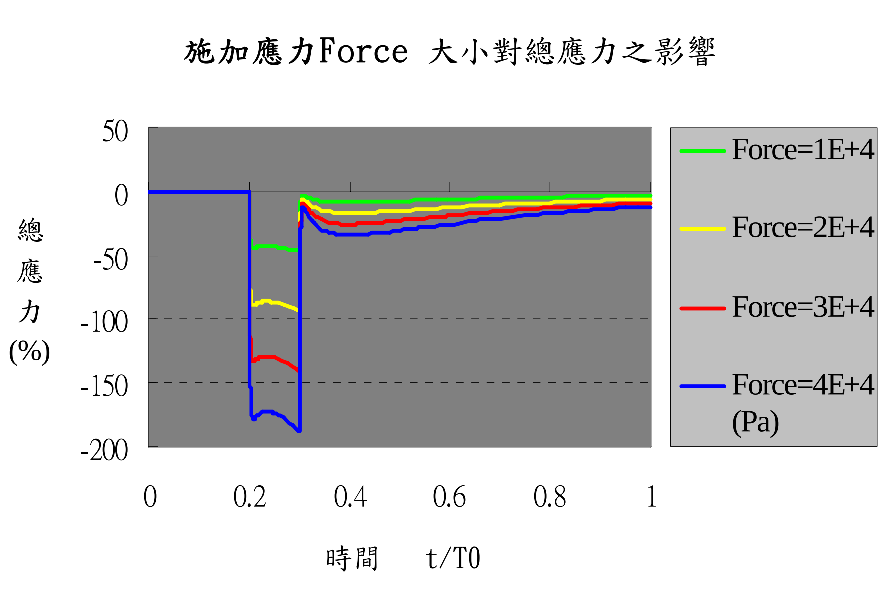
Damping coefficient

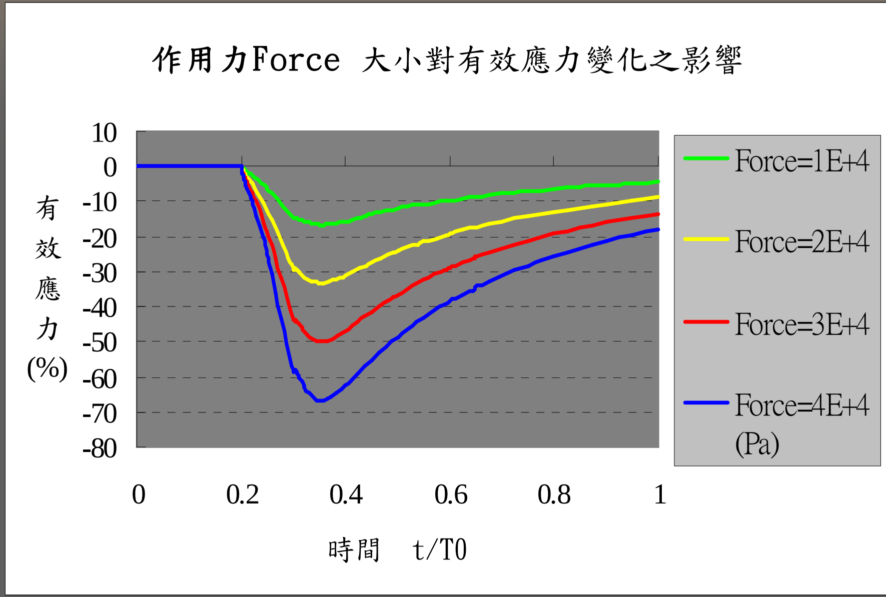


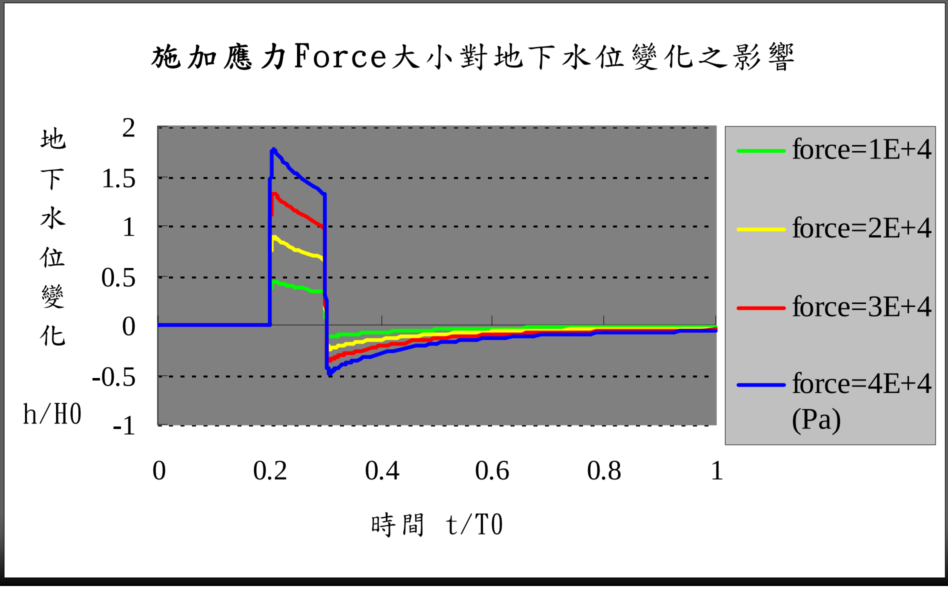




Amplitude of Excitations







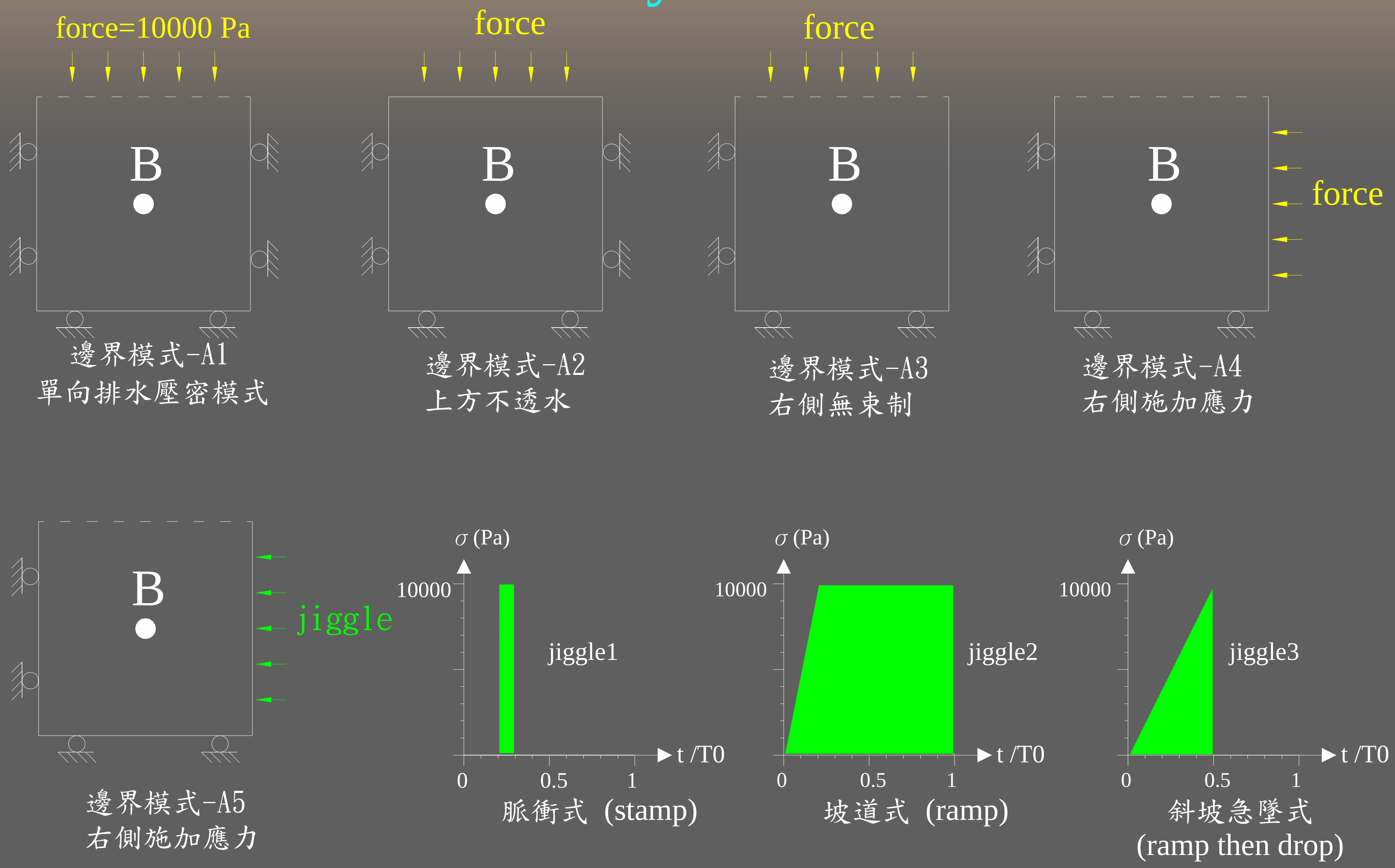
Summary of Parametric Study

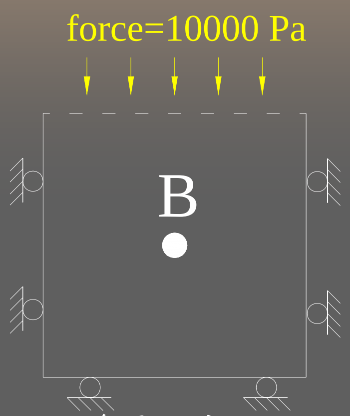
Parameters		Total Stress	Effective stress	Coseismic water level	Degree of influence
Hydraulic conductivity	↑	↓	↑	↓	☆☆☆
Poisson's ratio	↑	—	↑	↓	☆☆
Young's Modulus	↑	—	↑	↓	☆☆☆
Strain Amplification constant	↑	↑	↓	↑	☆☆☆
Damping coefficient	↑	↓	↓	↓	☆☆☆
Excitations	↑	↑	↑	↑	☆☆☆
Fluid compressibility	↑	—	—	—	×
Porosity	↑	—	—	—	×
Total density	↑	—	—	—	×

The effect of boundary condition

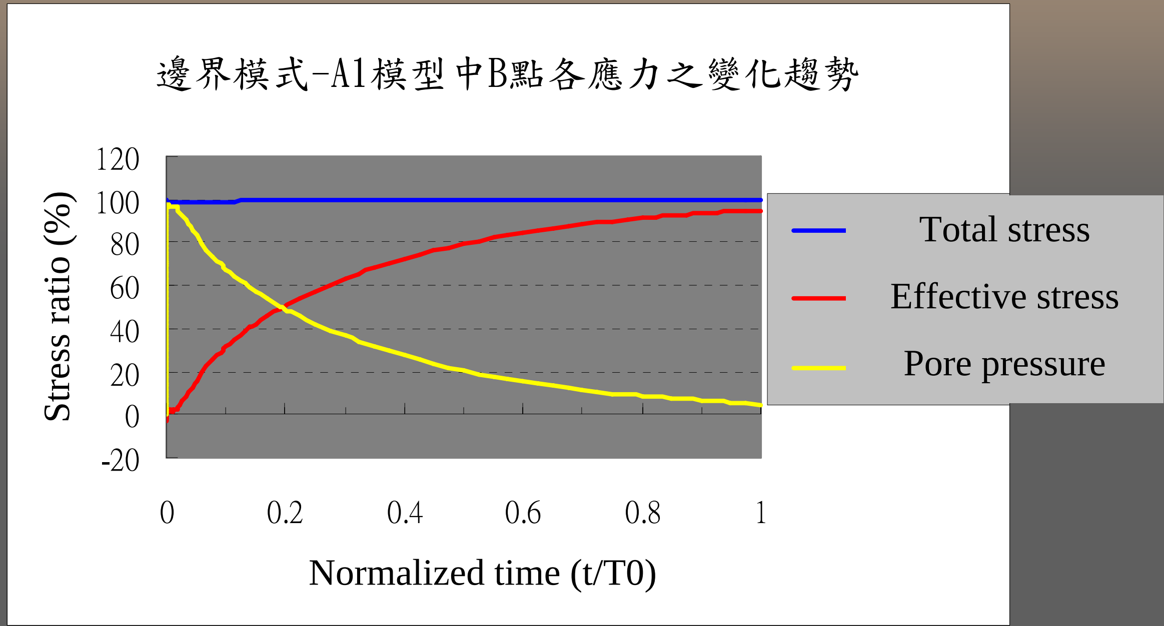
- Permeability of boundary
  - Drained boundary
  - Undrained boundary
- Constraint of boundary
  - Rigid boundary
  - Movable boundary
- Excitations on the boundary
  - Types of excitations

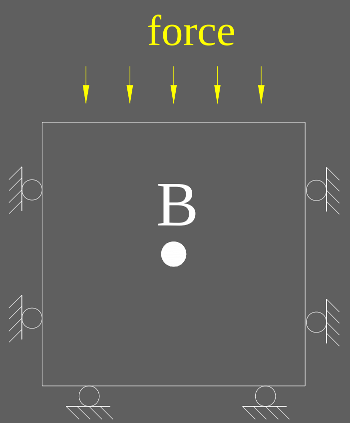
Boundary conditions



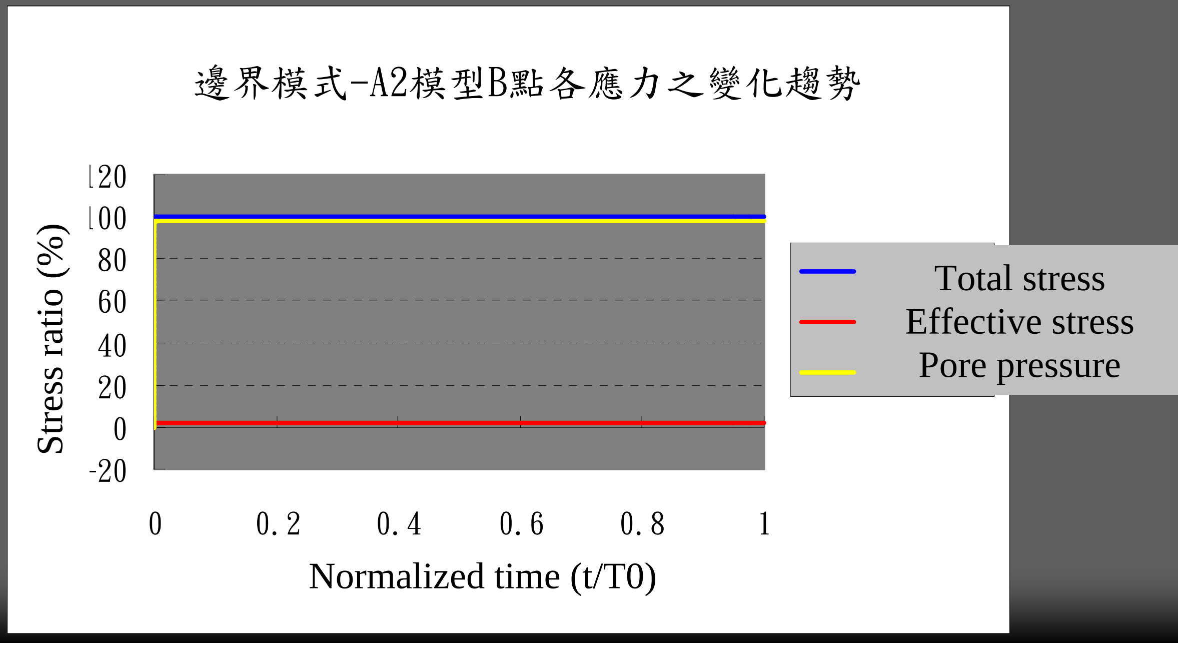


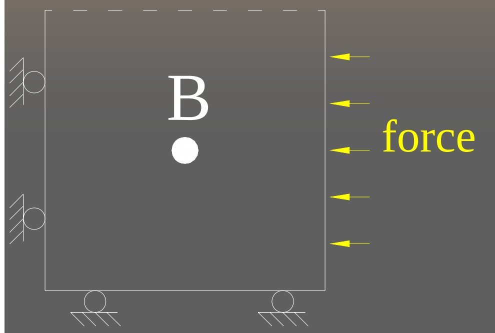
邊界模式-A1
單向排水壓密模式





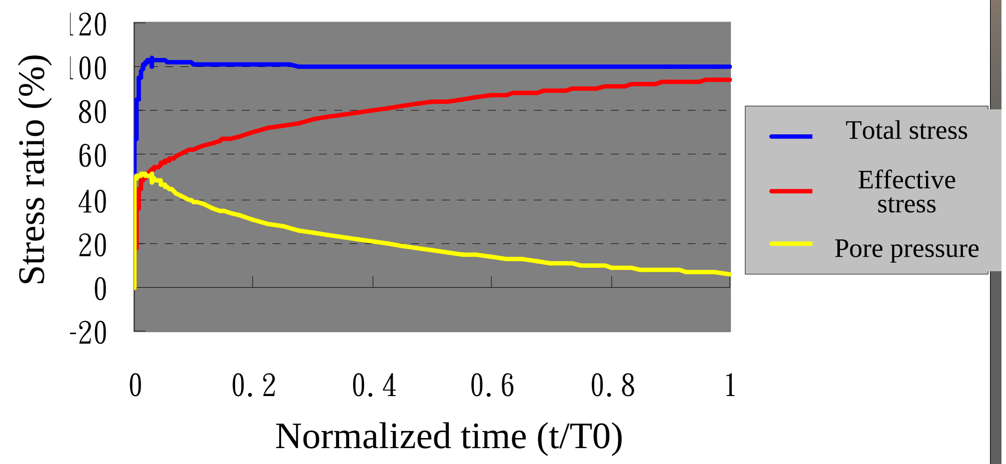
邊界模式-A2
上方不透水

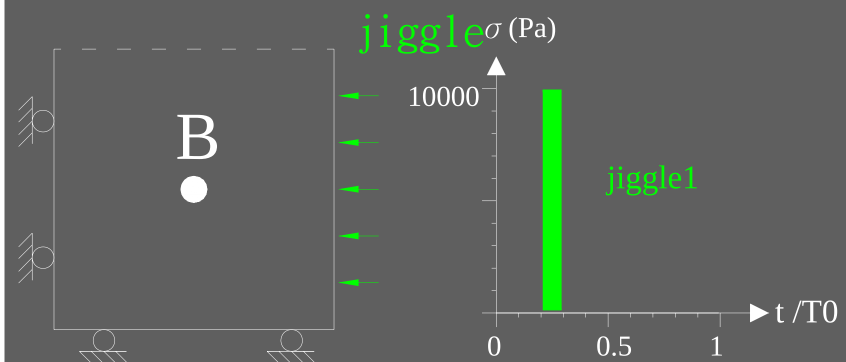




邊界模式-A4
右側施加應力

邊界模式-A4模型中B點各應力之變化趨勢

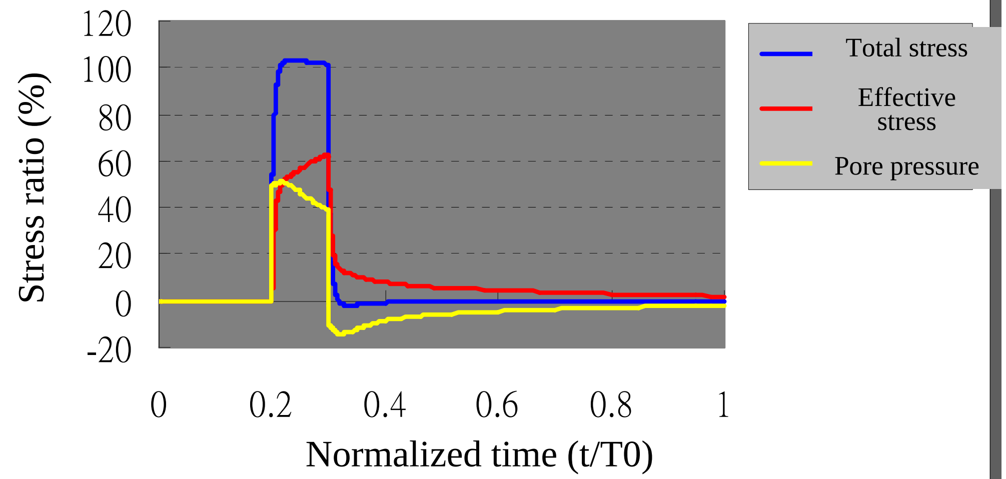


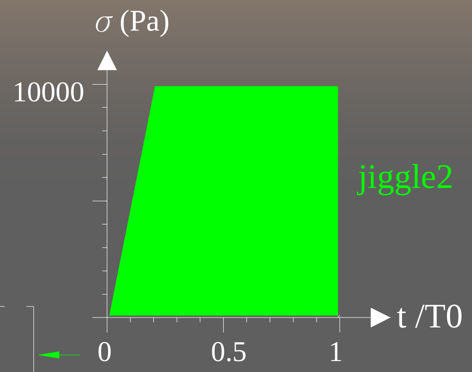


脈衝式 (stamp)

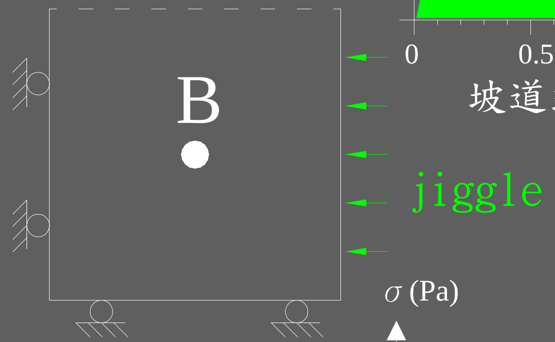
邊界模式-A5
右側施加應力

邊界模式-A5模型受jiggle1作用下模型B點各應力之變化趨勢

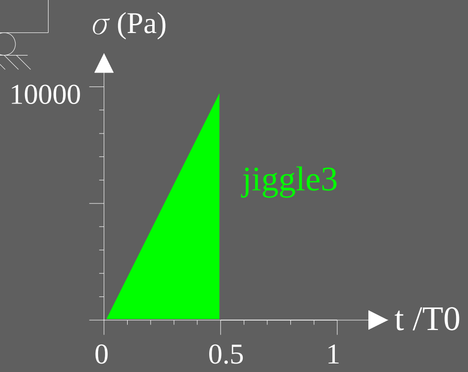




坡道式 (ramp)

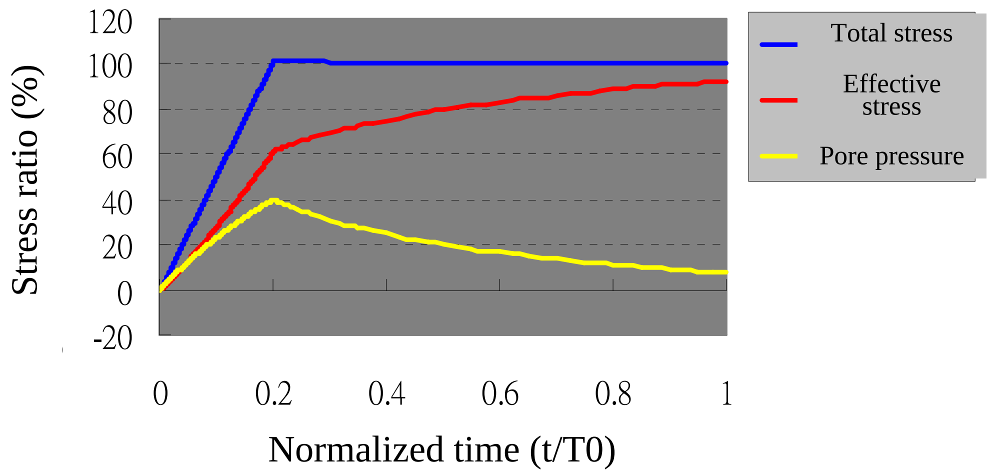


邊界模式-A5
右側施加應力

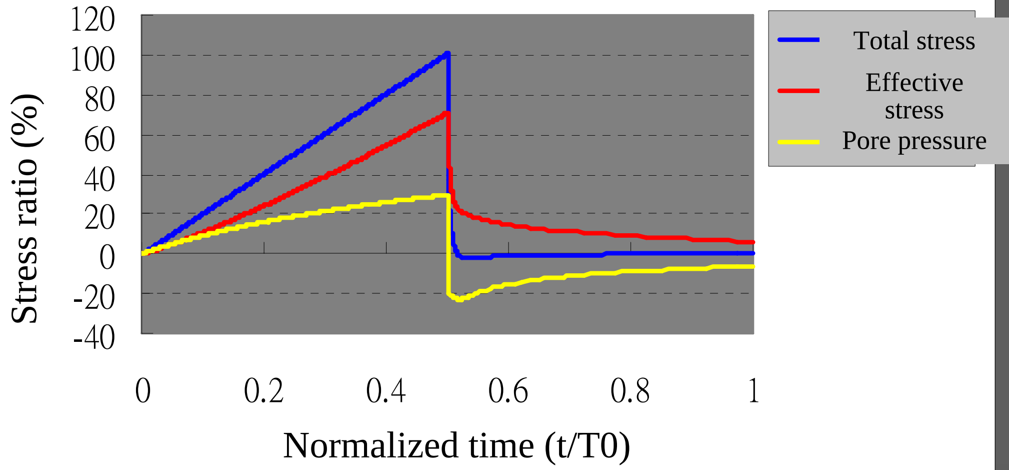


斜坡急墜式
(ramp then drop)

邊界模式-A5模型受jiggle2作用力下模式B點各應力之變化趨勢

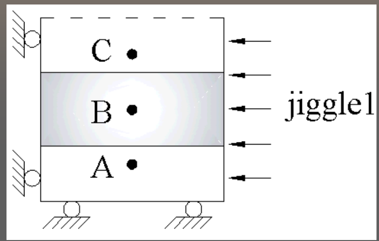


邊界模式-A5模型受jiggle3作用力下模型B點各應力之變化趨勢

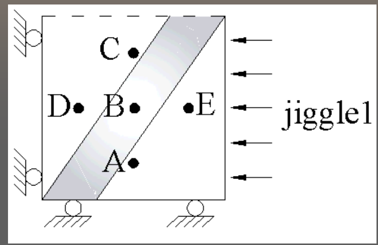


➲ Stratum Analysis

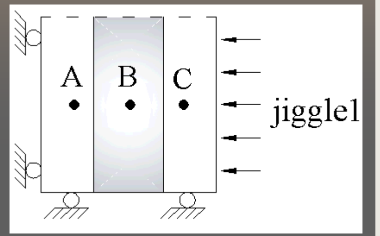
複層模式-A1



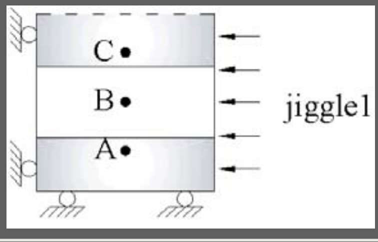
複層模式-A2



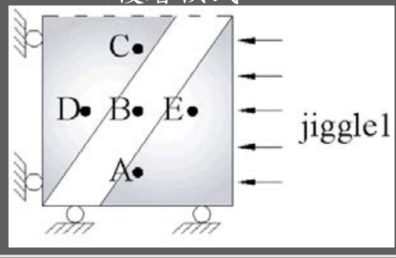
複層模式-A3



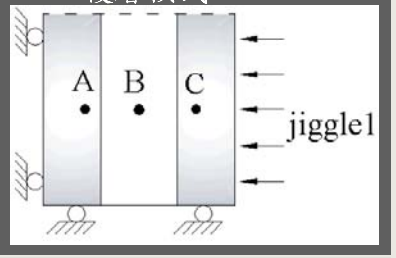
複層模式-B1



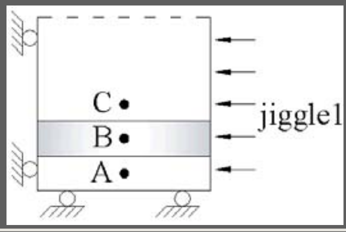
複層模式-B2



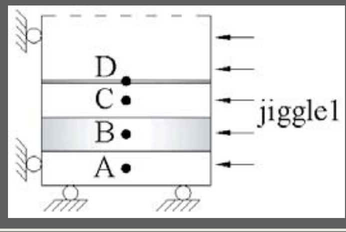
複層模式-B3



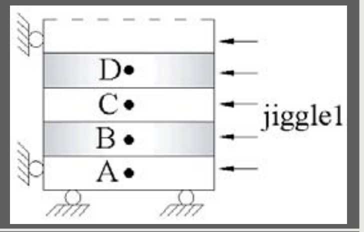
(自由含水層) 複層模式-C1



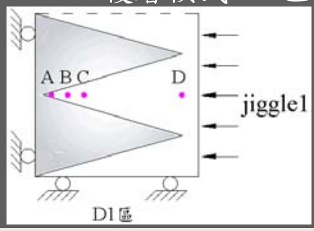
(部分侷限含水層) 複層模式-C2



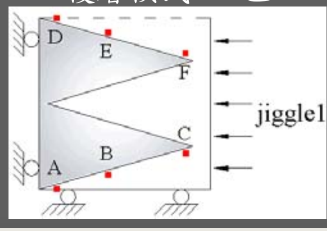
(侷限含水層) 複層模式-C3



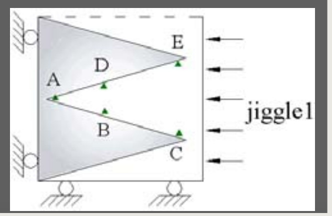
複層模式-D1區



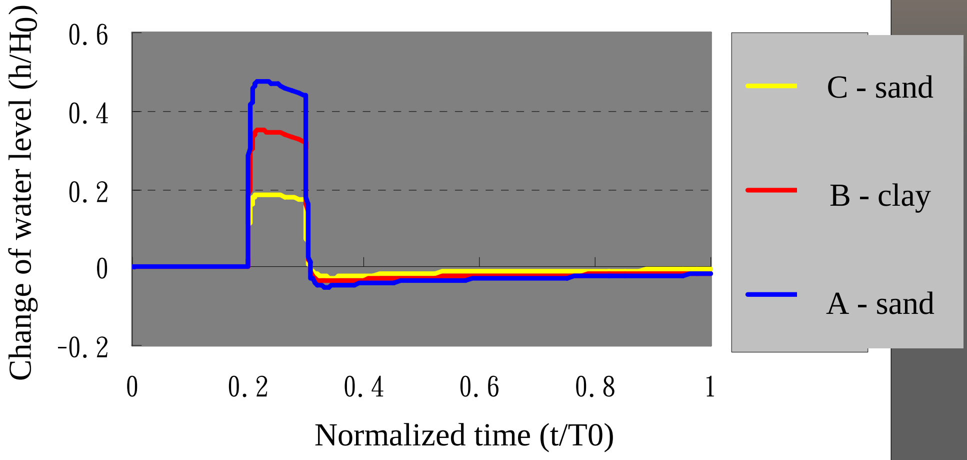
複層模式-D2區

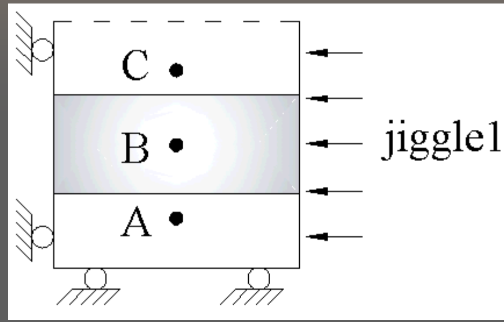


複層模式-D3區

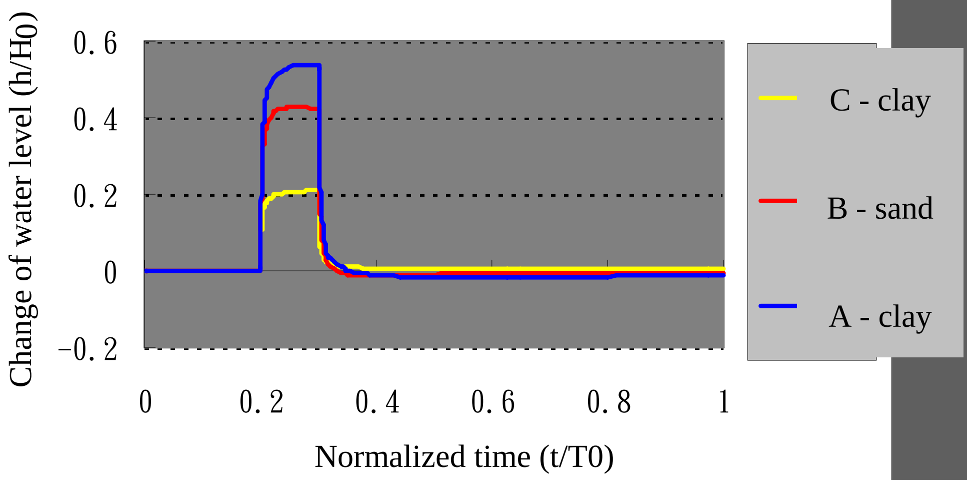


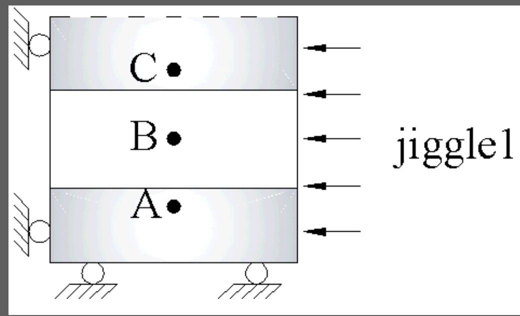
複層模式-A1模型受脈衝式震波作用
不同深度點其水位變化趨勢之比較

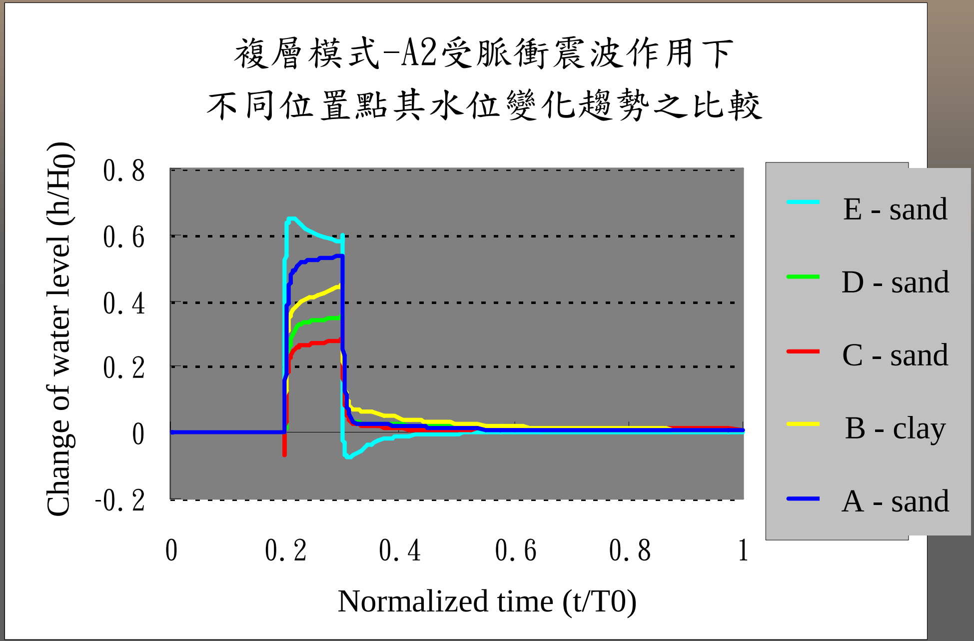


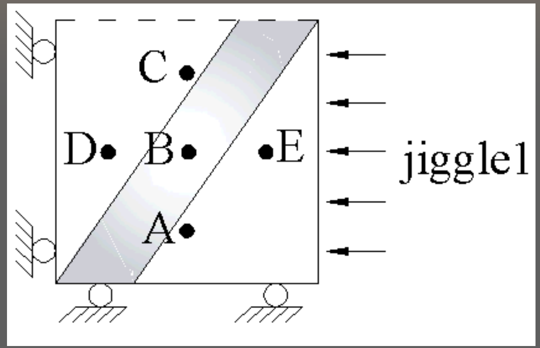


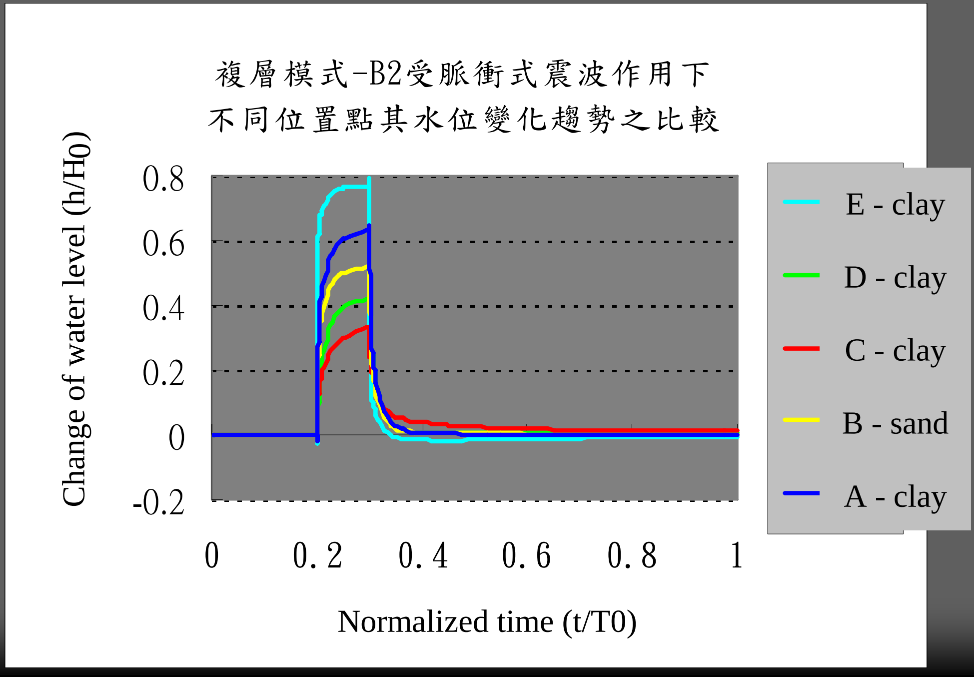
複層模式-B1受脈衝式震波作用下
不同深度點其水位變化趨勢之比較

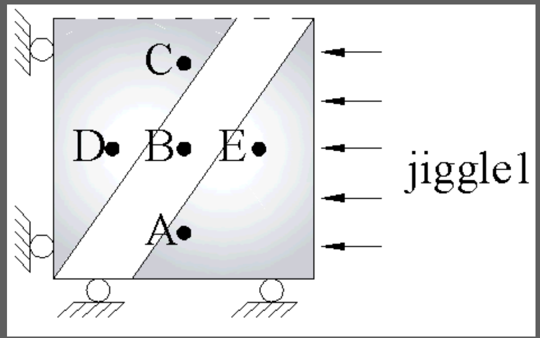


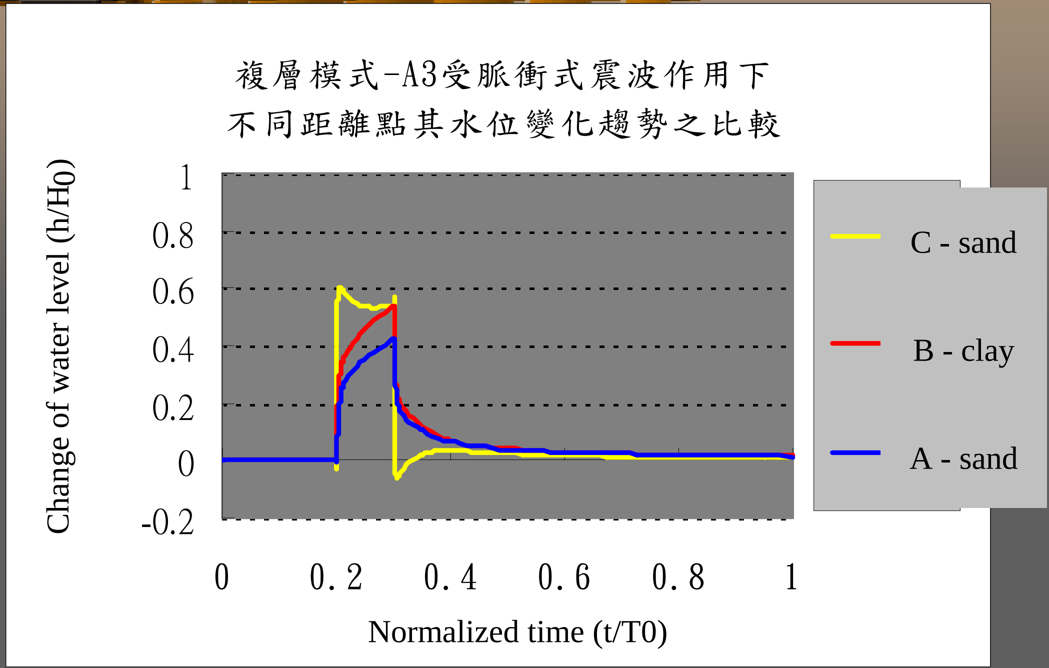


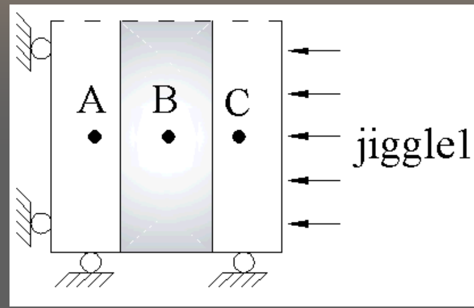

複層模式-A2受脈衝震波作用下
不同位置點其水位變化趨勢之比較
Change of water level (h/H0)
0.8
0.6
0.4
0.2
0
-0.2
0
0.2
0.4
0.6
0.8
1
Normalized time (t/T0)
E - sand
D - sand
C - sand
B - clay
A - sand

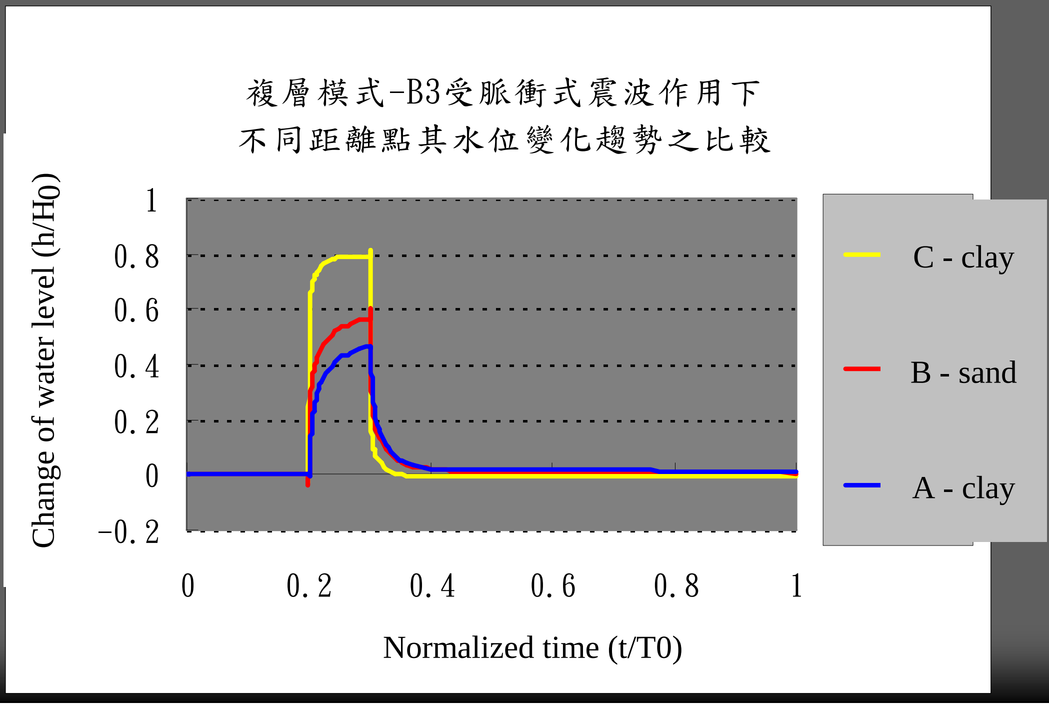

C
D
B
E
A
jiggle1

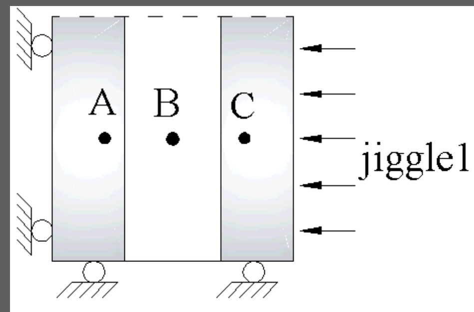

複層模式-B2受脈衝式震波作用下
不同位置點其水位變化趨勢之比較
Change of water level (h/H0)
0.8
0.6
0.4
0.2
0
-0.2
0
0.2
0.4
0.6
0.8
1
Normalized time (t/T0)
E - clay
D - clay
C - clay
B - sand
A - clay

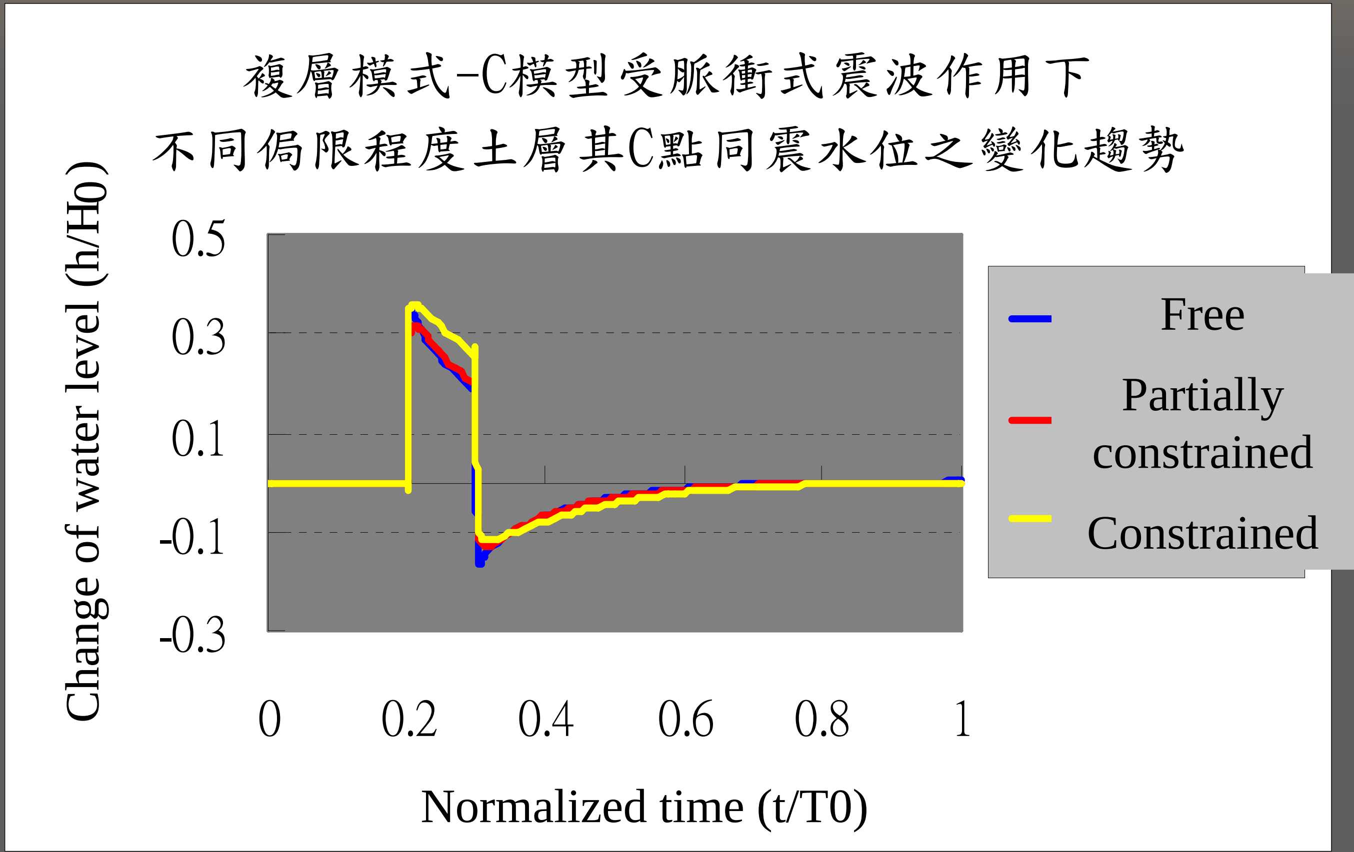

C
D
B
E
A
jiggle1

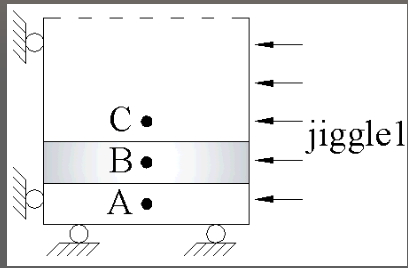

複層模式-A3受脈衝式震波作用下
不同距離點其水位變化趨勢之比較
Change of water level (h/H0)
1
0.8
0.6
0.4
0.2
0
-0.2
0
0.2
0.4
0.6
0.8
1
Normalized time (t/T0)
C - sand
B - clay
A - sand

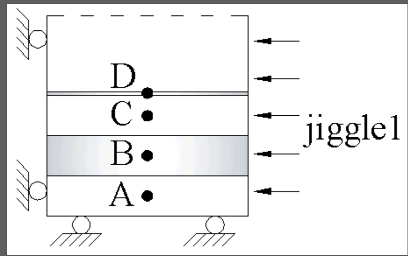

A
B
C
jiggle1

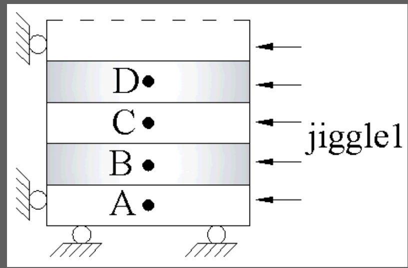

複層模式-B3受脈衝式震波作用下
不同距離點其水位變化趨勢之比較
Change of water level (h/H0)
1
0.8
0.6
0.4
0.2
0
-0.2
0
0.2
0.4
0.6
0.8
1
Normalized time (t/T0)
C - clay
B - sand
A - clay


A
B
C
jiggle1

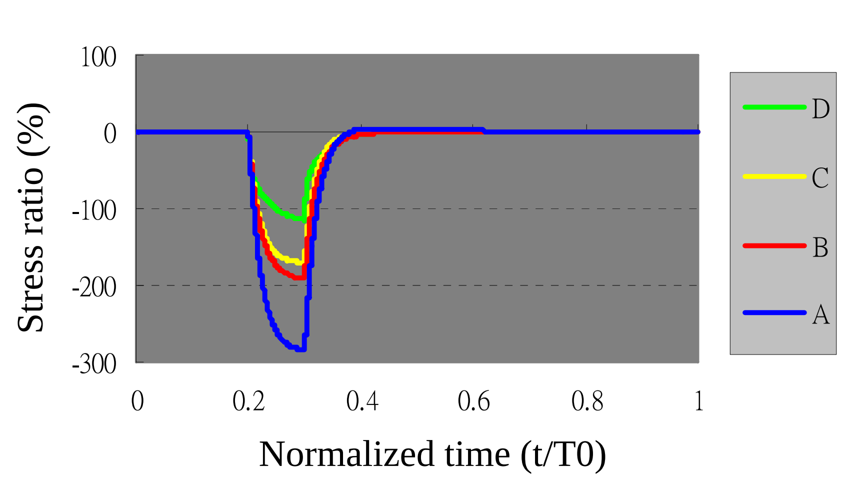

複層模式-C模型受脈衝式震波作用下
不同侷限程度土層其C點同震水位之變化趨勢
Change of water level (h/H0)
0.5
0.3
0.1
-0.1
-0.3
0
0.2
0.4
0.6
0.8
1
Normalized time (t/T0)
Free
Partially constrained
Constrained

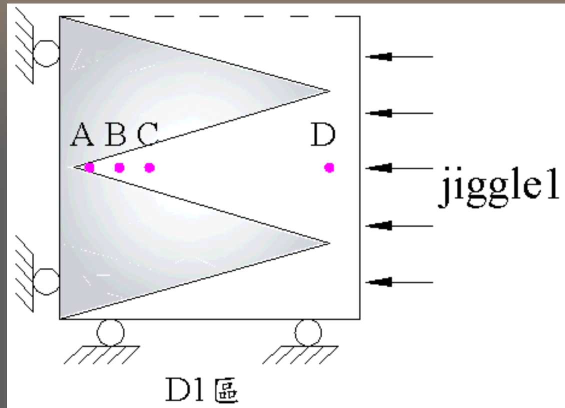

C
B
A
jiggle1


D
C
B
A
jiggle1

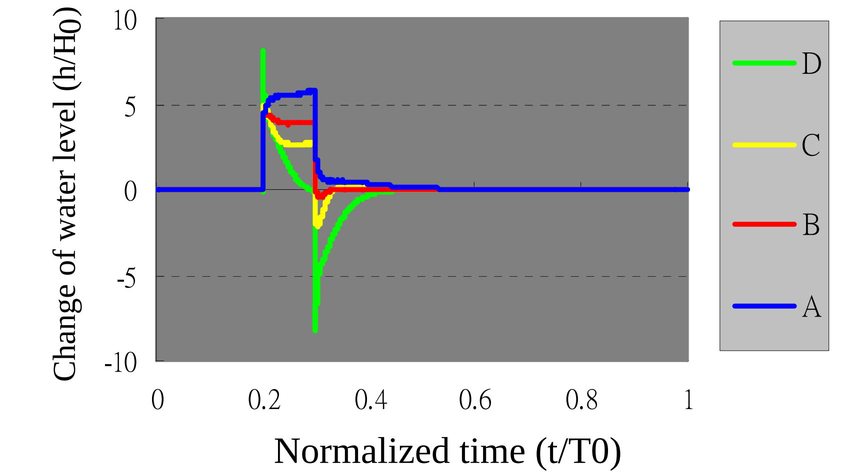

D
C
B
A
jiggle1

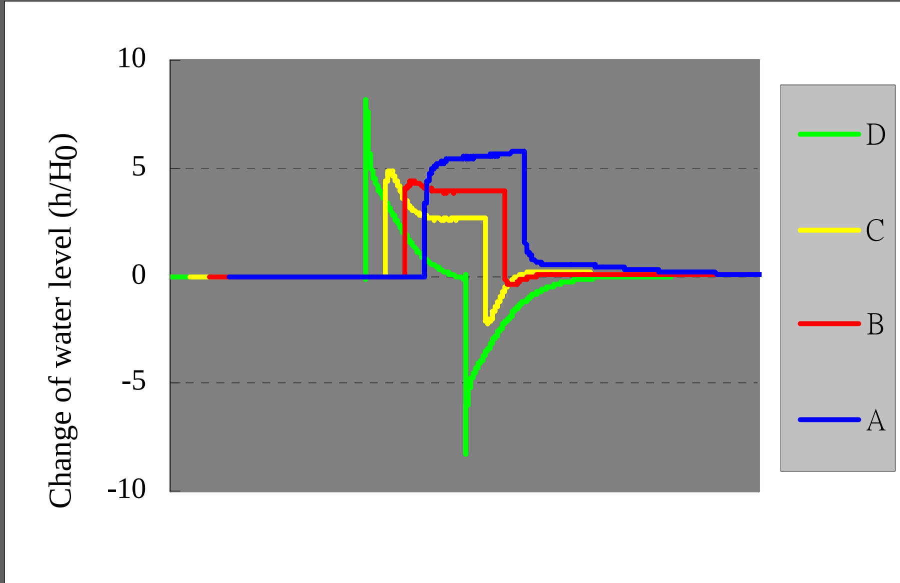
複層模式-D模型受脈衝式震波作用下
D1區各點有效應力之變化趨勢

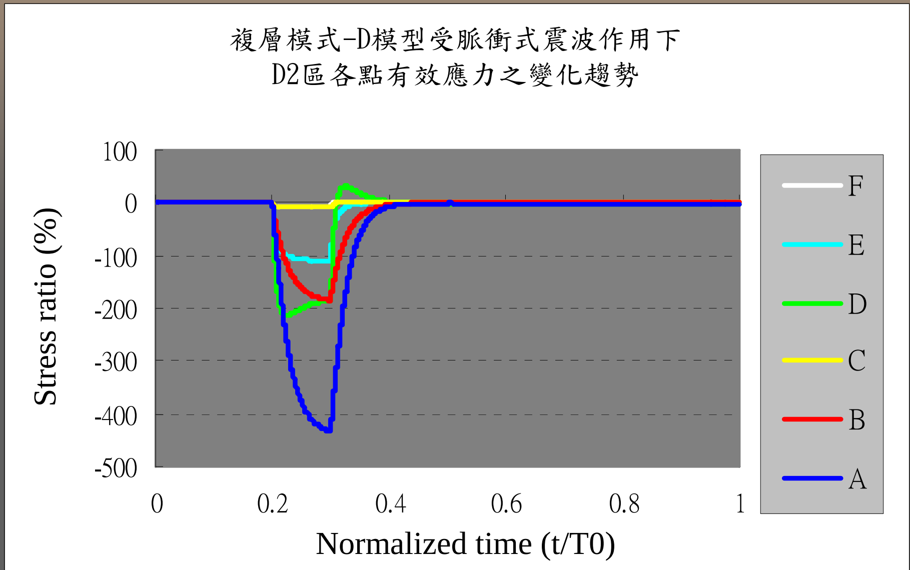


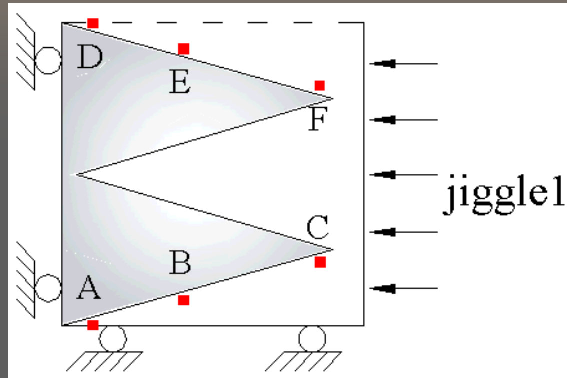


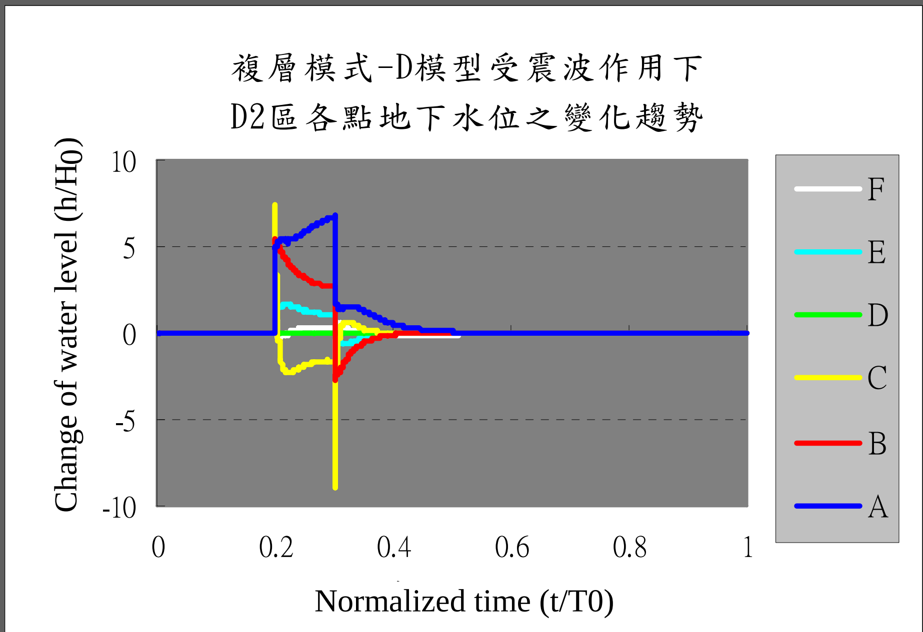
複層模式-D模型受脈衝式震波作用下
D1區各點地下水位之變化趨勢

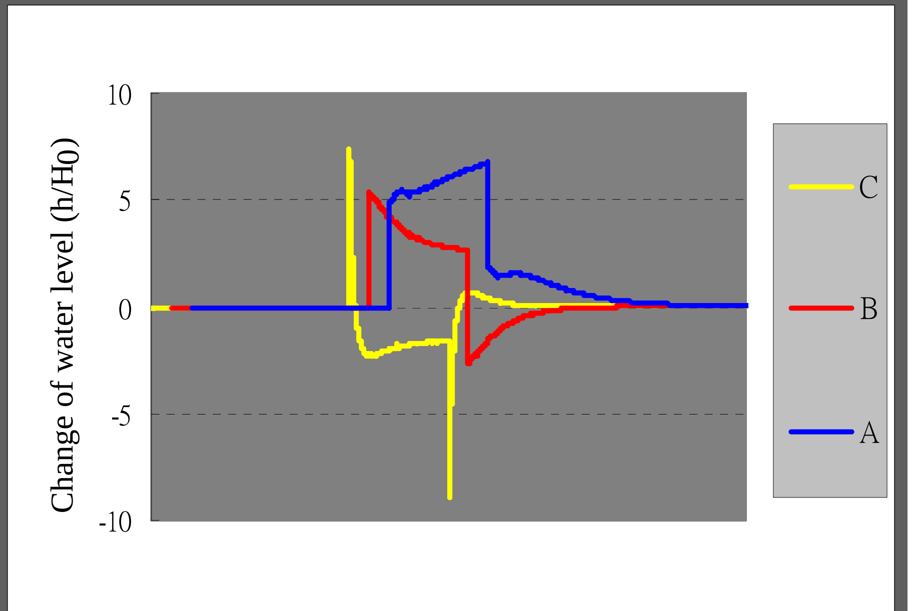


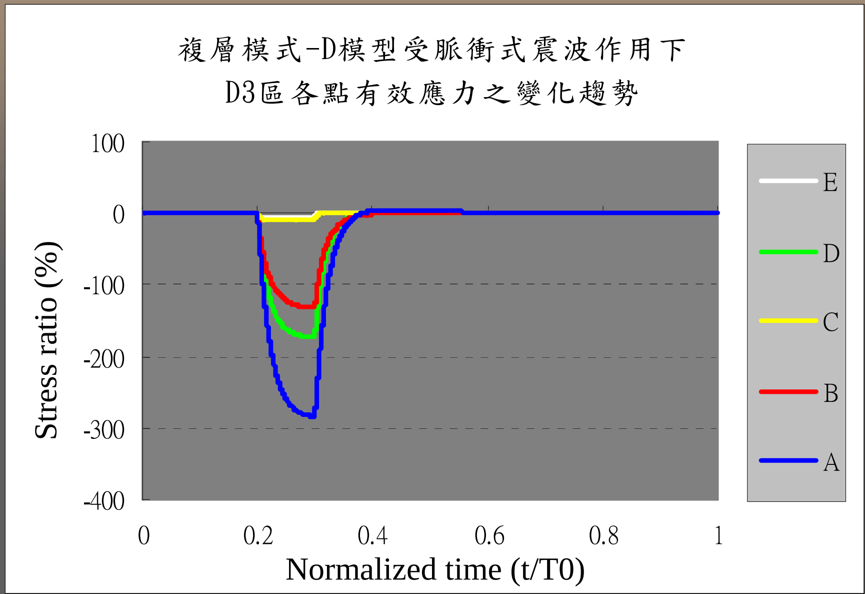


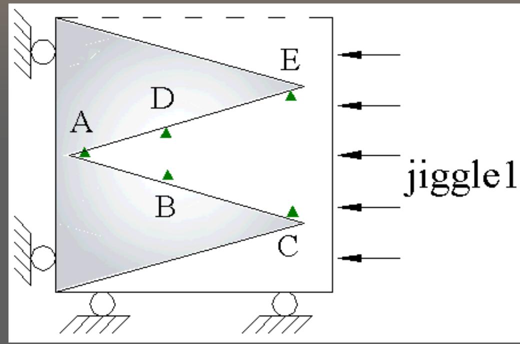

複層模式-D模型受脈衝式震波作用下
D2區各點有效應力之變化趨勢
Stress ratio (%)
100
0
-100
-200
-300
-400
-500
0
0.2
0.4
0.6
0.8
1
Normalized time (t/T0)
F
E
D
C
B
A

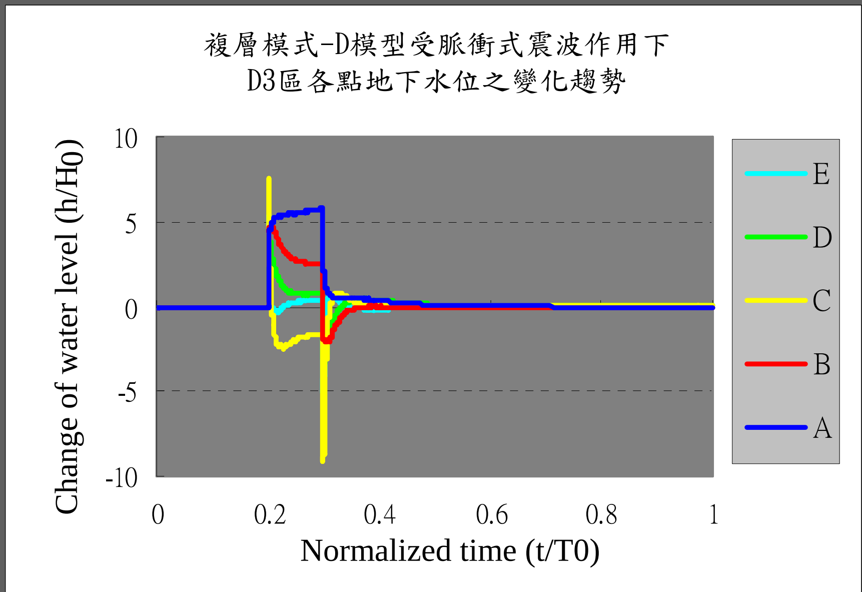

D
E
F
C
B
A
jiggle1

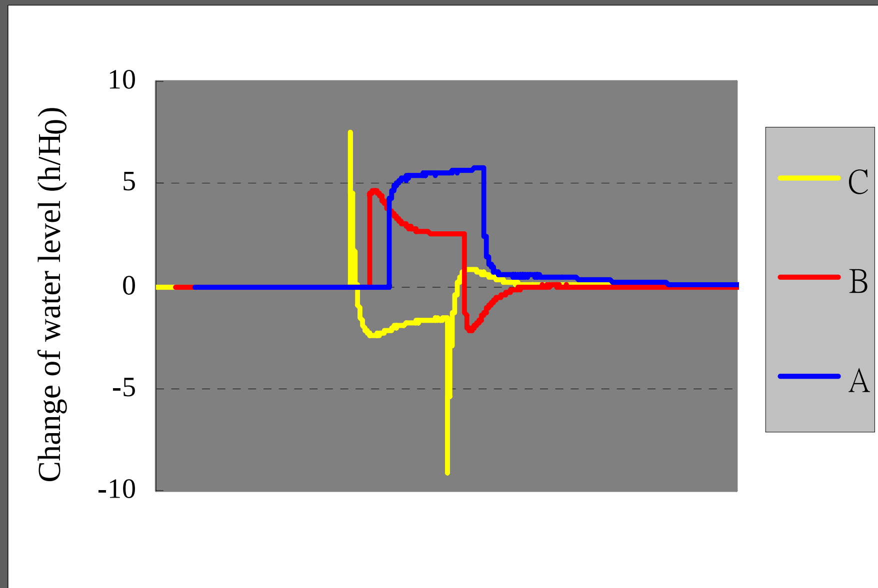

複層模式-D模型受震波作用下
D2區各點地下水位之變化趨勢
Change of water level (h/H0)
10
5
0
-5
-10
0
0.2
0.4
0.6
0.8
1
Normalized time (t/T0)
F
E
D
C
B
A


Change of water level (h/H0)
10
5
0
-5
-10
C
B
A

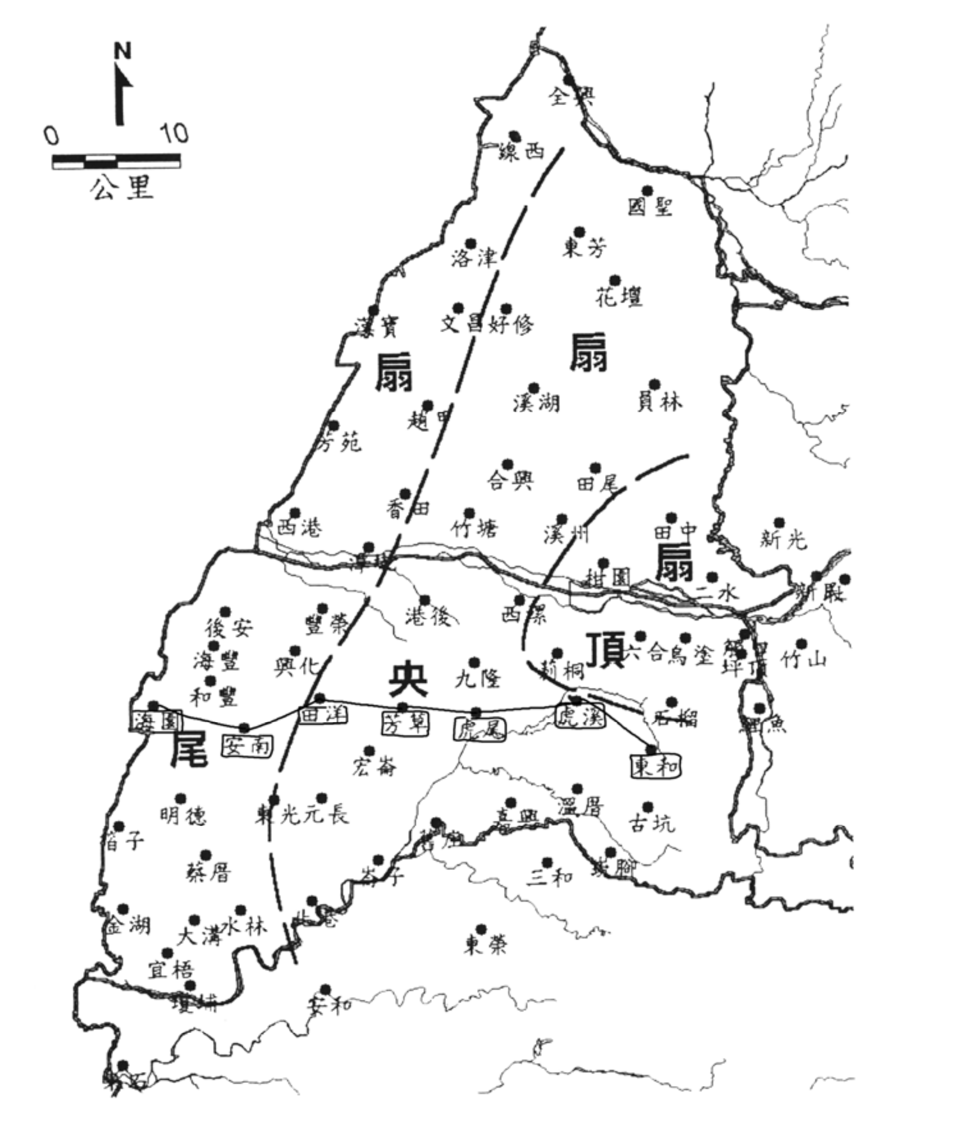

複層模式-D模型受脈衝式震波作用下
D3區各點有效應力之變化趨勢
Stress ratio (%)
100
0
-100
-200
-300
-400
0
0.2
0.4
0.6
0.8
1
Normalized time (t/T0)
E
D
C
B
A

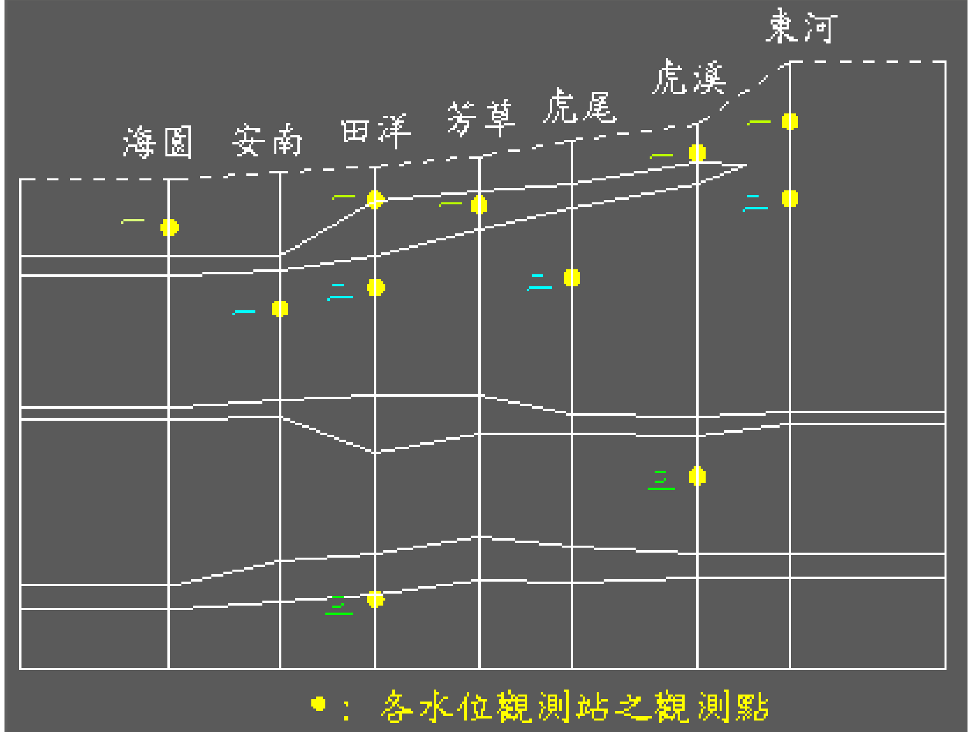

E
D
A
B
C
jiggle1

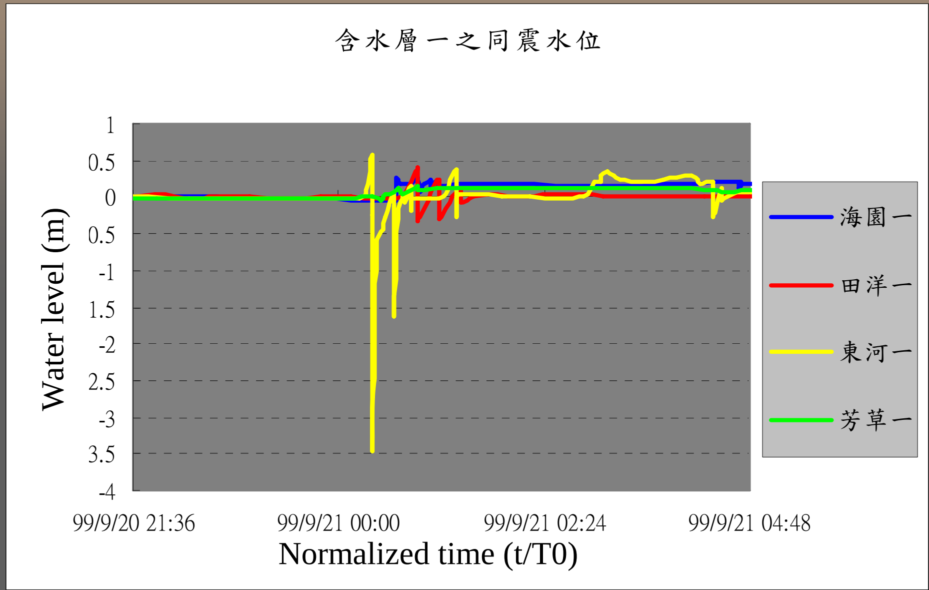

複層模式-D模型受脈衝式震波作用下
D3區各點地下水位之變化趨勢
Change of water level (h/H0)
10
5
0
-5
-10
0
0.2
0.4
0.6
0.8
1
Normalized time (t/T0)
E
D
C
B
A

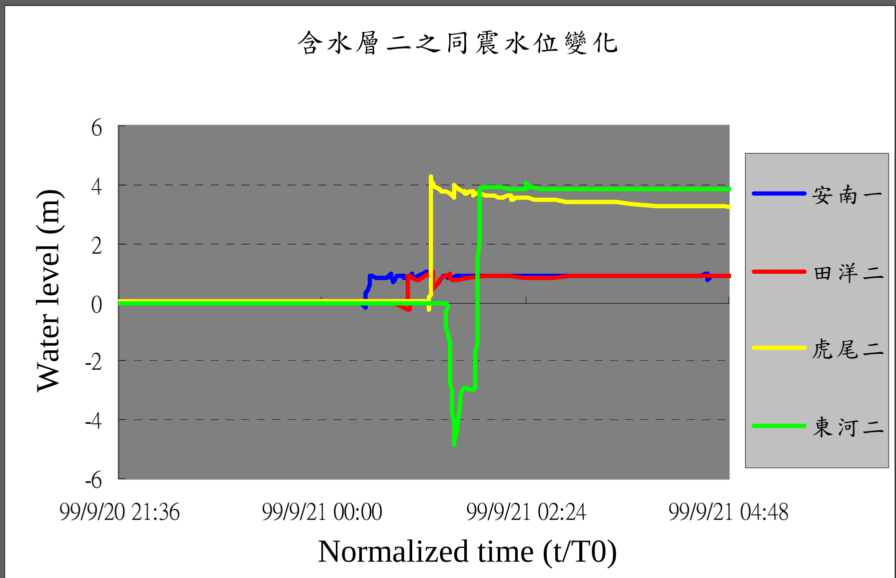

Change of water level (h/H0)
10
5
0
-5
-10
C
B
A

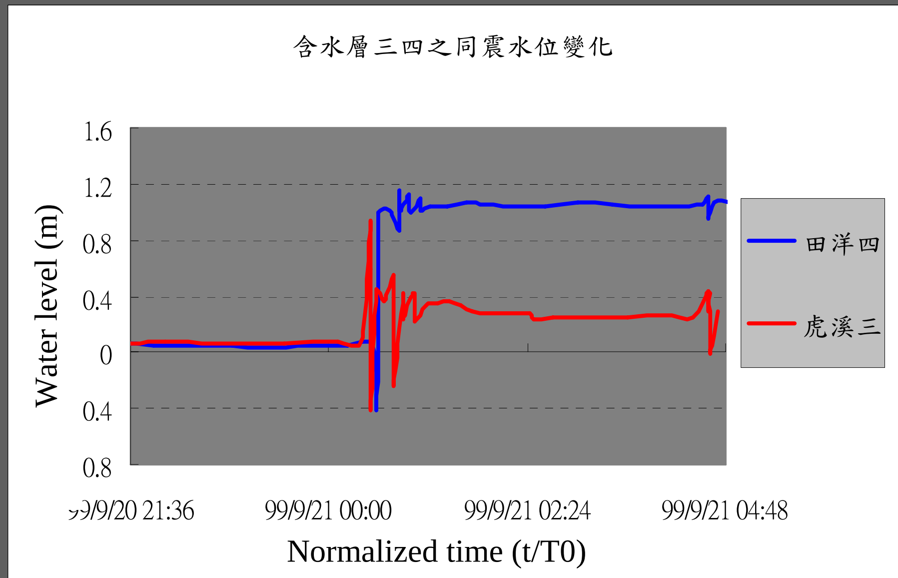
Case Study

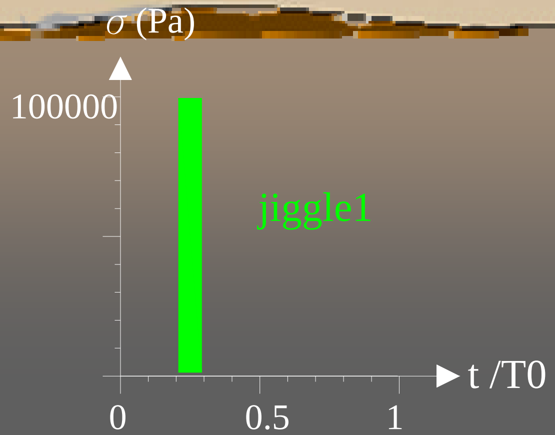




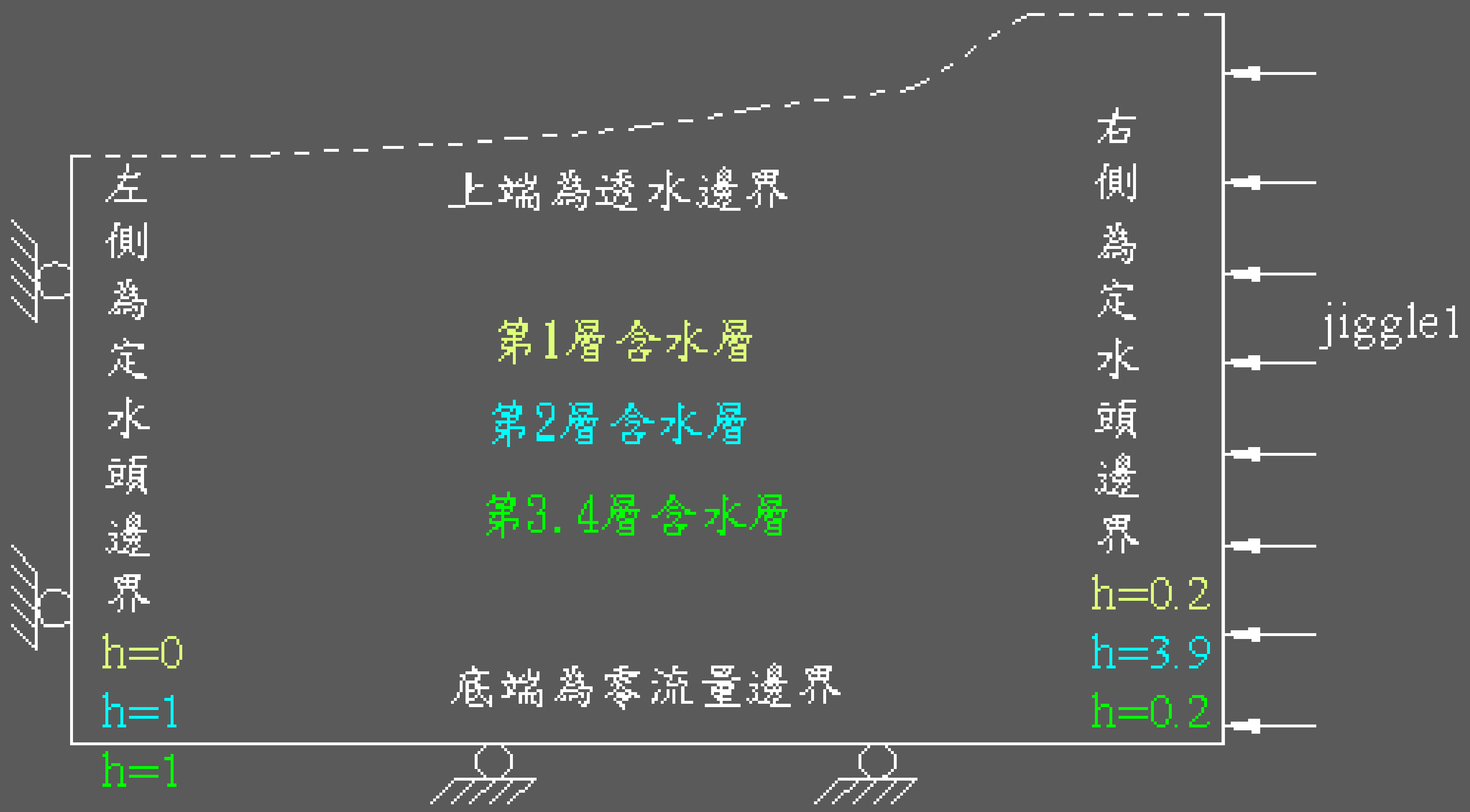

含水層一之同震水位
Water level (m)
1
0.5
0
0.5
-1
1.5
-2
2.5
-3
3.5
-4
99/9/20 21:36
99/9/21 00:00
99/9/21 02:24
99/9/21 04:48
Normalized time (t/T0)
海園一
田洋一
東河一
芳草一


含水層二之同震水位變化
Water level (m)
6
4
2
0
-2
-4
-6
99/9/20 21:36
99/9/21 00:00
99/9/21 02:24
99/9/21 04:48
Normalized time (t/T0)
安南一
田洋二
虎尾二
東河二


含水層三四之同震水位變化
Water level (m)
1.6
1.2
0.8
0.4
0
0.4
0.8
99/9/21 00:00
99/9/21 02:24
99/9/21 04:48
Normalized time (t/T0)
田洋四
虎溪三


σ (Pa)
100000
jiggle1
t /T0
0
0.5
1

脈衝式 (stamp)

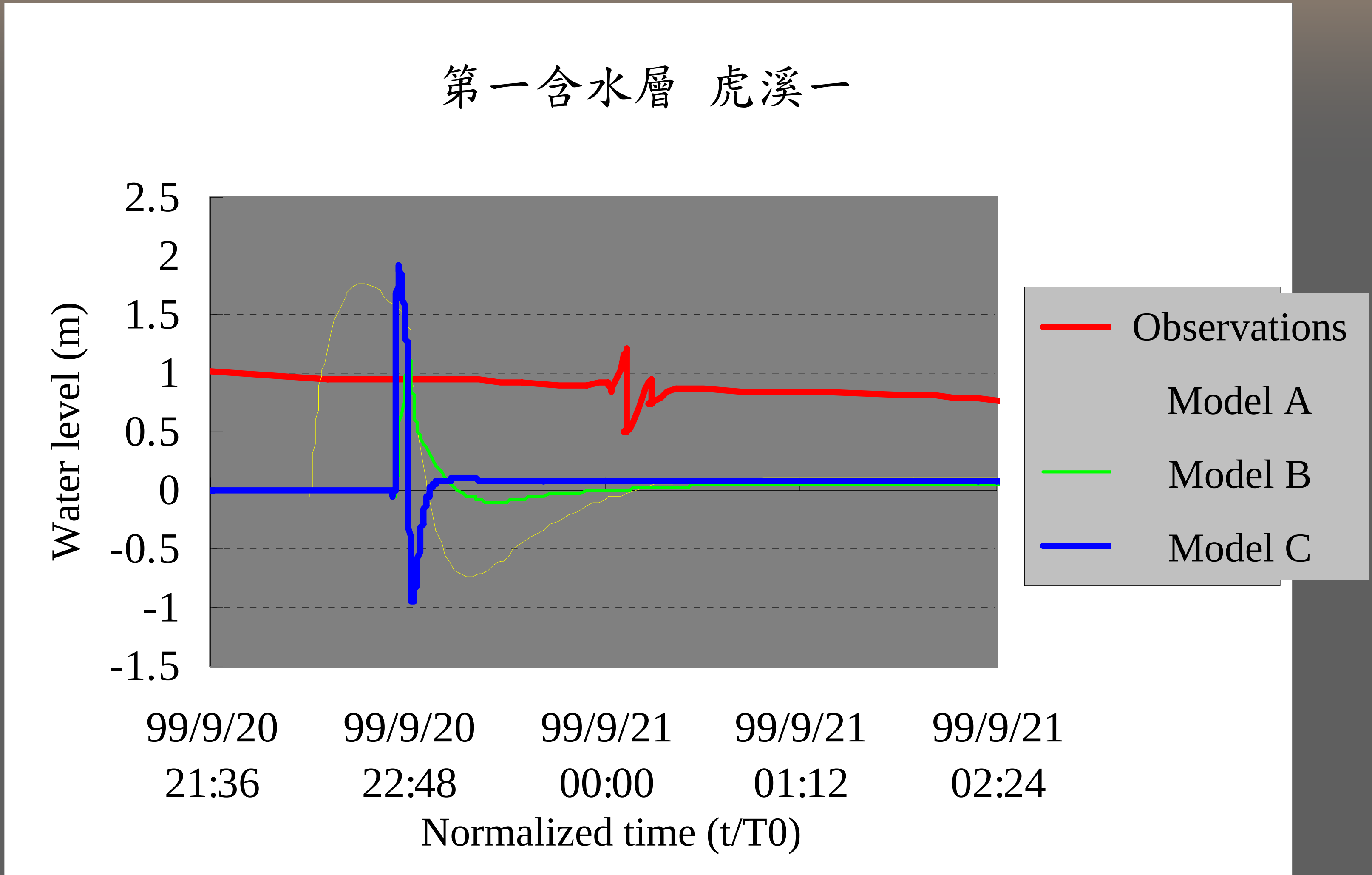

上端為透水邊界
左側為定水頭邊界
h=0
h=1
h=1
第1層含水層
第2層含水層
第3.4層含水層
底端為零流量邊界
右側為定水頭邊界
h=0.2
h=3.9
h=0.2
jiggle1

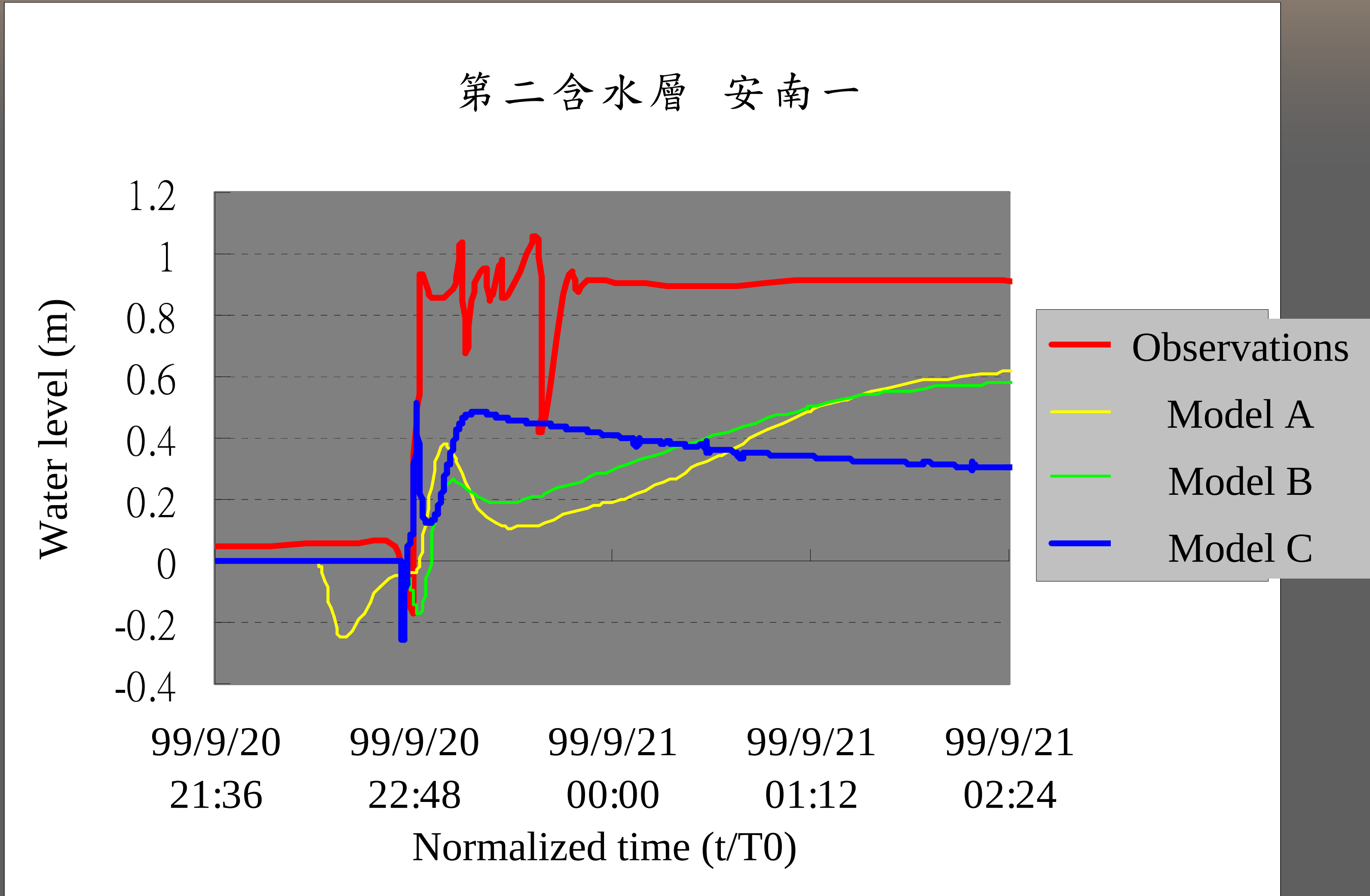
Parameter	Name	Aquifer	Aquitard
Hydrology	Hydraulic conductivity (m/sec) Fluid compressibility(Pa^{-1}) Unit weight of fluid(N/m^3)	$k_{xx}=k_{zz}$=1e-4 β_f =4.4e-10 γ_w =9810	$k_{xx}=k_{zz}$=1e-7 β_f =4.4e-10 γ_w =9810
Material	Porosity (-) Young’s modulus（Pa） Total Density（kg/m^3） Poisson’s ratio (-)	**n** =0.375 E=1e8 ρ =2100 ν =0.25	**n** =0.55 E=1e7 ρ =1870 ν =0.3
Dynamics	Damping constant（Pa.s/m^2） Strain amplification（-）	ζ =1e+9 χ =5	

Mode A: jiggle1 ($0.05<t<0.1$) , ζ =1e+9

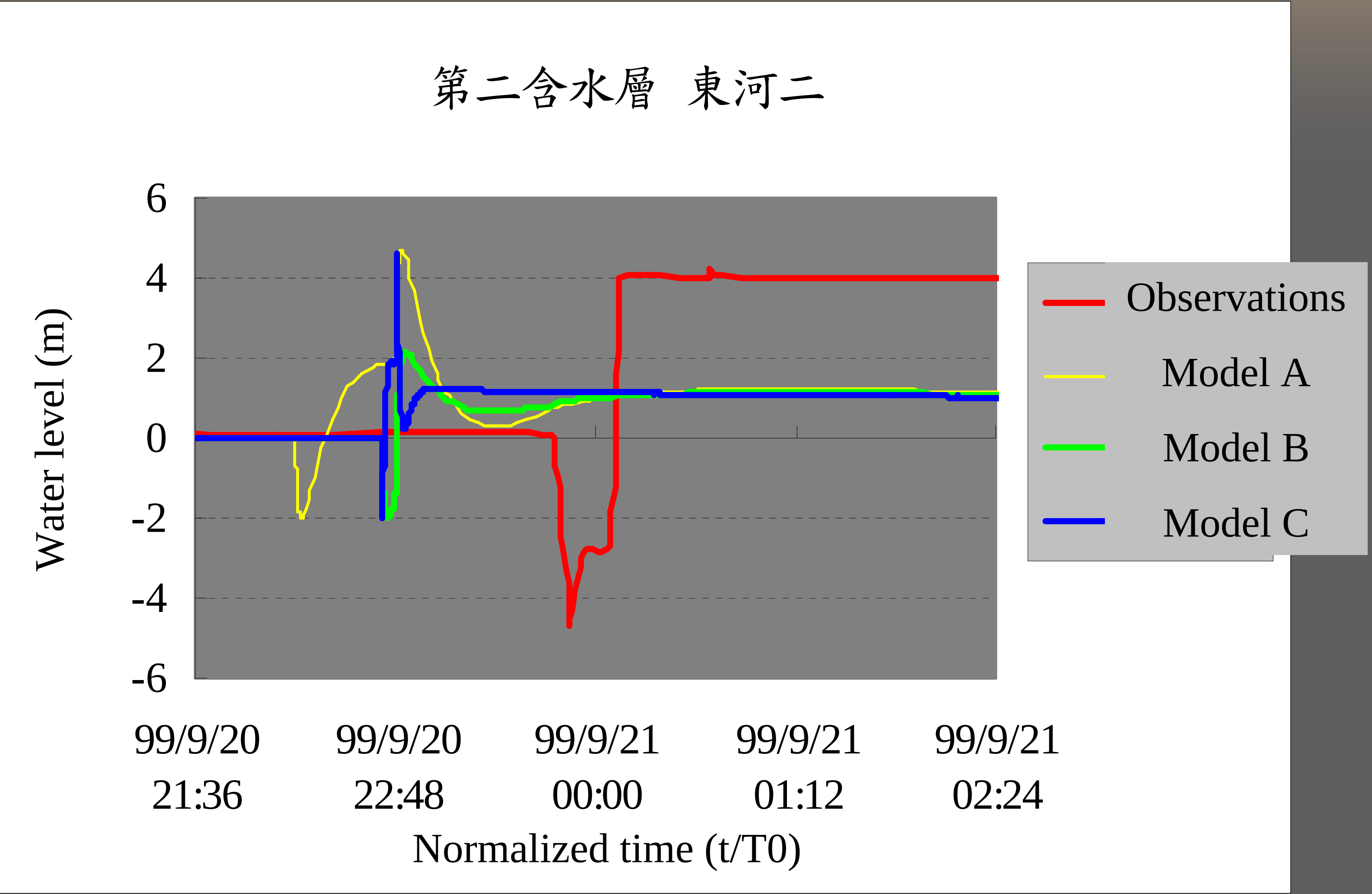
Mode B: jiggle1 ($0.09<t<0.1$) , ζ =1e+9

Mode C: jiggle1 ($0.09<t<0.1$) , ζ =1e+8

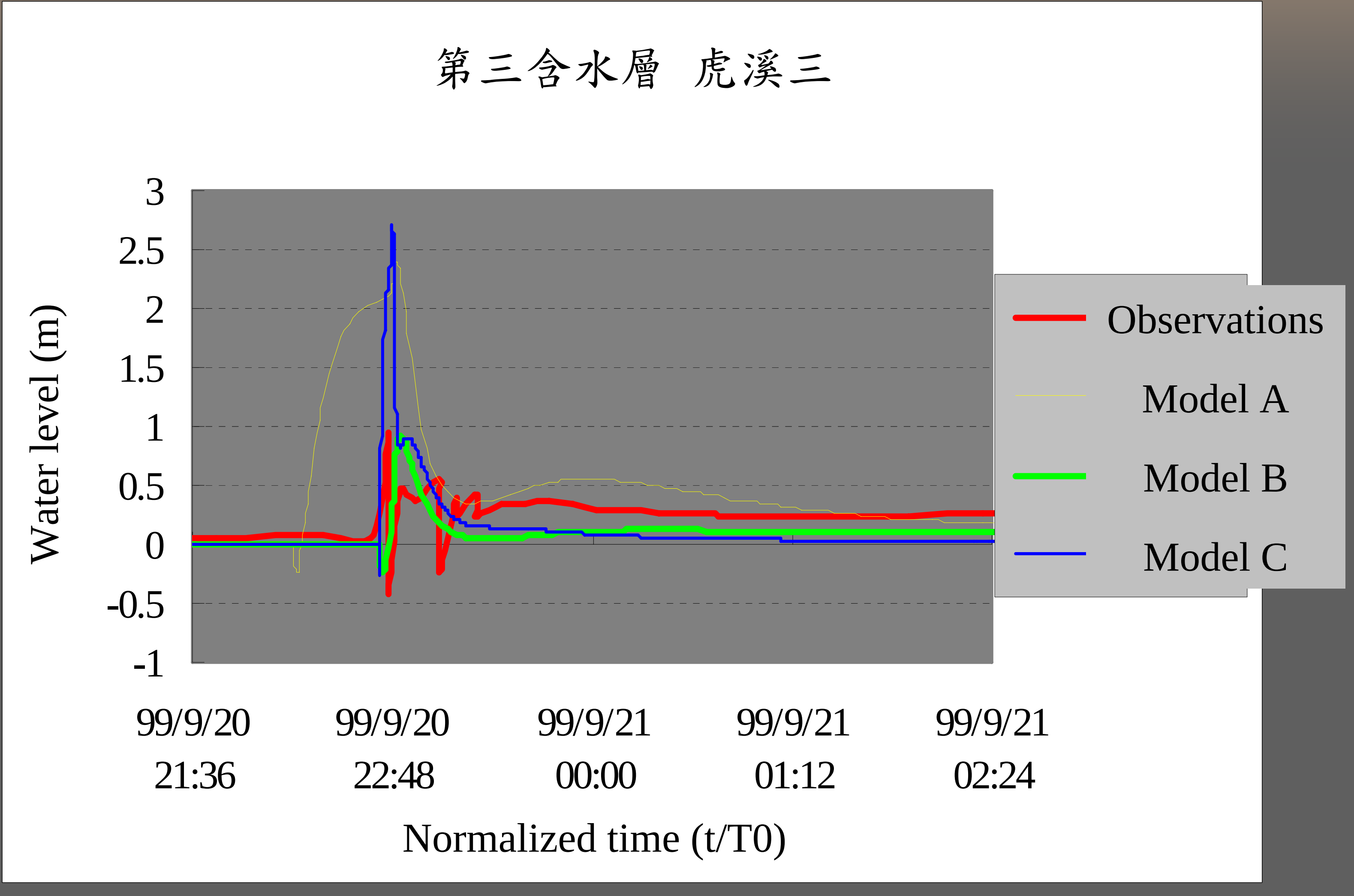

第一含水層 虎溪一
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
Water level (m)
99/9/20 21:36
99/9/20 22:48
99/9/21 00:00
99/9/21 01:12
99/9/21 02:24
Normalized time (t/T0)
Observations
Model A
Model B
Model C



第二含水層 安南一
Water level (m)
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
99/9/20 21:36
99/9/20 22:48
99/9/21 00:00
99/9/21 01:12
99/9/21 02:24
Normalized time (t/T0)
Observations
Model A
Model B
Model C



第二含水層 東河二
Water level (m)
6
4
2
0
-2
-4
-6
99/9/20 21:36
99/9/20 22:48
99/9/21 00:00
99/9/21 01:12
99/9/21 02:24
Normalized time (t/T0)
Observations
Model A
Model B
Model C



第三含水層 虎溪三
Water level (m)
3
2.5
2
1.5
1
0.5
0
-0.5
-1
99/9/20 21:36
99/9/20 22:48
99/9/21 00:00
99/9/21 01:12
99/9/21 02:24
Normalized time (t/T0)
Observations
Model A
Model B
Model C

Calibration

分析點	模式A			模式B			模式C		
(水位觀測站)	觀測值	分析值	修正係數	觀測值	分析值	修正係數	觀測值	分析值	修正係數
海園一	-0.058	0.239	-0.243	-0.058	0.090	-0.648	-0.058	0.317	-0.183
田洋一	0.413	-0.147	-2.814	0.413	-0.138	-3.002	0.413	-0.151	-2.744
芳草一	-0.043	0.438	-0.099	-0.043	0.122	-0.355	-0.043	0.692	-0.063
虎溪一	0.379	1.770	0.214	0.379	-0.046	-8.238	0.379	1.912	0.198
東河一	0.532	-0.715	-0.743	0.532	-0.716	-0.742	0.532	-0.741	-0.717
安南一	-0.229	-0.250	0.917	-0.229	-0.174	1.320	-0.229	-0.257	0.894
田洋二	-0.270	1.164	-0.232	-0.270	0.270	-1.000	-0.270	1.442	-0.187
虎尾二	-0.322	1.697	-0.190	-0.322	0.406	-0.793	-0.322	2.074	-0.155
東河二	-4.773	-1.968	2.425	-4.773	-1.970	2.424	-4.773	-2.010	2.375
虎溪三	0.848	-0.244	-3.468	0.848	-0.244	-3.468	0.848	-0.257	-3.296
田洋三	-0.363	0.521	-0.697	-0.363	0.278	-1.307	-0.363	0.493	-0.737

Conclusions

- Modified dynamic poroelastic theory
- Numerical study
  - Sensitivity study
  - Effect of boundary condition
  - Stratum layer analysis
  - Case Study

When my students see this picture, happy summer is about over!