Time-domain decoupled poroelastic equations for saturated porous media

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The strong coupling of applied stress and pore fluid pressure, known as poroelasticity, is relevant to a number of applied problems arising in hydrogeology and reservoir engineering. The standard theory of poroelastic behavior in a homogeneous, isotropic, elastic porous medium saturated by a viscous, compressible fluid is due to Biot, who derived a pair of coupled partial differential equations that accurately predict the existence of two independent dilatational (compressional) wave motions, corresponding to in-phase and out-of-phase displacements of the solid and fluid phases, respectively. The Biot equations can be decoupled exactly after Fourier transformation to the frequency domain, but the resulting pair of Helmholtz equations cannot be converted to partial differential equations in the time domain and, therefore, closed-form analytical solutions of these equations in space and time variables cannot be obtained. In this paper we show that the decoupled Helmholtz equations can in fact be transformed to two independent partial differential equations in the time domain if the wave excitation frequency is very small as compared to a critical frequency equal to the kinematic viscosity of the pore fluid divided by the permeability of the porous medium. The partial differential equations found are a propagating wave equation and a dissipative wave equation, for which closed-form solutions are known under a variety of initial and boundary conditions. Numerical calculations indicate that the magnitude of the critical frequency for representative sedimentary materials containing either water or a nonaqueous phase liquid is in the kHz-MHz range, which is generally above the seismic band of frequencies. Therefore, the two partial differential equations obtained should be accurate for modeling elastic wave phenomena in fluid-saturated porous media under typical low-frequency conditions applicable to hydrogeological problems, e.g. simulation of groundwater level fluctuation induced by earthquakes.